The Effects of Word-Learning Biases on Children's Concept of Angle

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Despite evidence that young children are sensitive to differences in angle measure, older students frequently struggle to grasp this important mathematical concept. When making judgments about the size of angles, children often rely on erroneous dimensions such as the length of the angles' sides. The present study tested the possibility that this misconception stems from the whole-object word-learning bias by providing a subset of children with a separate label to refer to the whole angle figure. Thirty preschoolers (M = 4.86 years, SD = .53) were tested with a pretest–training–posttest design. At pretest, children showed evidence of the whole-object misconception. After training, children who were given a novel-word label for the whole object improved significantly more than those trained on the meaning of "angle" alone.

Whether they are formed by the boundaries of a room, the edges of an object, or the lines of a two-dimensional drawing, angles are an important feature of our everyday visual experience. A significant body of research suggests that a system for representing angle is present early in development (Cohen & Younger, 1984; Lourenco & Huttenlocher, 2008; Schwartz & Day, 1979) and may even be accessible to newborn infants (Slater, Mattock, Brown, & Bremner, 1991). By the age of 4, children are capable of comparing angles across multiple two-dimensional figures (Izard & Spelke, 2009) as well as across two-dimensional and three-dimensional figures (Izard, O'Donnell, & Spelke, 2014). Spelke and colleagues have even proposed that children's ability to perceive angles is a component of their initial system for representing objects and their features (Spelke, Lee, & Izard, 2010). Taken together, these findings demonstrate that young children are not only adept at visually processing angles in their environment but are also capable of making decisions based on those percepts.

In view of this early-developing sensitivity to angle, it is surprising that older students display a great deal of difficulty when learning about the concept of "angle" in school (Clements & Battista, 1992; Mitchelmore & White, 2000). Most notably, children in elementary and even middle school often focus on irrelevant properties such as the length of an angle's sides, the area within the sides, or the absolute distance between the sides when making judgments about the size of angles (Clements & Battista, 1989; Lindquist & Kouba, 1989). When presented with two angles, one that is formed by long lines and one that is formed by short lines, these children often mistakenly believe that the angle formed by longer lines is larger, regardless of the measure of the angles. This misconception is not only pervasive but also surprisingly stable over several years of schooling (Lehrer, Jenkins, & Osana, 1998). On the surface, these findings seem to be at odds with findings that children perceive even subtle variations in angles by 4 years of age.

However, an important difference between the tasks used to measure toddlers' sensitivity to angle and those used to measure school-aged children's understanding of angle is the degree to which the tasks explicitly ask about "size" and "angle." For instance, Izard and Spelke (2009) used a Deviant Detection Paradigm in which young children were presented with six angle figures and asked to "pick the item that looked different." On key trials, all the figures differed in terms of the length of their sides but only one differed in terms of its angle. In

contrast, Lehrer et al. (1998) presented students with two angles and had them to make explicit judgments by asking them to "select the larger angle." Why do older children continue to struggle with this second type of task even though much younger children succeed on the first type of task?

Previously, researchers have attributed students' errors to a difficulty separating angle from other dimensions of a visual stimulus, such as length or area (Clements & Battista, 1989). Such explanations draw largely on the Van Hiele model of geometric reasoning (Van Hiele, 1986), which states that children are initially incapable of viewing shapes in terms of their component properties (e.g., sides and angles) and instead rely solely on a holistic processing approach. Further support for this possibility comes from findings that while older children readily sort objects on the basis of a single dimension (e.g., color), children below age 5 typically focus on the objects holistically and sort them in terms of overall similarity (e.g., grouping objects of similar color and shape; Smith, 1983, 1984) or overall size (Smith, 1985).

Importantly, such an account does not imply that children are unable to perceive angle altogether but rather that they may be unable to disembed angles from the overall figures that contain them. In other words, Smith (1989) proposed that while object perception is likely constructed from analyses of separate dimensions, children's ultimate object representations might not preserve dimensional independence. Therefore, children could still succeed on Izard and Spelke's (2009) Deviant Detection task by picking the holistically deviant figure—the figure that differed from the other figures in terms of its overall shape (both side length and angle)—rather than by noticing that all of the figures aside from one were identical along a single dimension. Consistent with this possibility, Izard and Spelke found that young children made more errors when line length was varied among the figures rather than held constant. Moreover, when children made errors, they were most likely to select the figures that fell on the extremes in terms of the lengths of their sides (either the shortest or the longest sides), a pattern that is consistent with the kinds of difficulties displayed by older children.

However, there are reasons to believe that a difficulty separating perceptual dimensions is not the sole source of children's angle misconceptions. Most strikingly, while students continue to base judgments of angle measure on erroneous dimensions well into middle school (e.g., Lehrer et al., 1998)

children generally can use a single dimension to make similarity and magnitude judgments in other domains by the age of 5 (Smith, 1984, 1985). Moreover, while the ability to perceive separable dimensions is necessary to correctly answer the question, "Which is the bigger angle?" it is not sufficient. A child must also understand that the label "angle" refers to a specific separable dimension and not to other features of the figure or the figure itself. In the present study, we investigated this linguistic reference problem as another possible source of children's confusion regarding angles.

In particular, language learners' interpretations of unfamiliar words are influenced by the whole-object bias—the tendency to interpret a label for an unfamiliar object as referring to that object as a whole and not to the object's parts or properties (e.g., Hollich, Golinkoff, & Hirsh-Pasek, 2007; Landau, Smith, & Jones, 1988; Markman & Hutchinson, 1984). This general bias may be exacerbated in the case of angle by the fact that "angle" is commonly used interchangeably to refer to the measure of rotation between two intersecting line segments *or* to the angle figure as a whole (Angle, n.d.). Thus, even if children are capable of understanding that the length of an angle figure's sides has no effect on its measure of rotation, they still may be led astray by incorrectly mapping "angle" to the entire angle figure. By this account, when presented with a canonical angle figure (two line segments meeting at a point), children may prefer a whole-object interpretation of "angle" and thus believe a "bigger angle" is a larger figure and not a figure with a larger measure of rotation. Such an explanation is consistent with the findings that children tend to rely on side length, distance, or area, since these properties all increase with the size of the overall figure. Thus, it is possible that children's difficulty learning about angles in a formal school setting is the result of a word-mapping problem rather than a purely conceptual problem dictated by cognitive constraints, that is, difficulty separating dimensions.

The language learning literature also provides evidence that under the right conditions, even very young children are capable of mapping words to component properties of objects. In particular, children are more likely to map a novel label onto a part or property of an object when they already know a separate label for the whole object (Hall, Waxman, & Hurwitz, 1993; Markman & Wachtel, 1988). Based on this assumption, known as mutual exclusivity, if children are first taught a separate label for the whole angle figure, they should be less likely to map "angle" onto the incorrect dimension

(e.g., overall size) and more likely to map "angle" onto the correct dimension (measure of rotation). If true, this would implicate the whole-object bias as a contributing factor in children's misconceptions regarding the size of angles and would also provide a potential avenue for improving children's understanding of angle.

The present study was carried out to test the possibility that the whole-object bias contributes to children's difficulty with angles. All children were given a brief training session in which they were taught that angle size is determined by the measure of rotation and not other dimensions such as the size of the overall figure. Alone, this type of training is very similar to what students encounter in the classroom. Previous research demonstrating the difficulty with which children overcome angle-related misconceptions led us to predict that this type of training would be largely ineffective. Children in the experimental condition were given this same training but were also taught about the angle figure as a whole, which we labeled a "toma." On the basis of our hypothesis that the whole-object bias does contribute to students' difficulty understanding angles, we predicted that children in the experimental condition would learn more about the meaning of *angle* than those given training on the meaning of angle alone. However, if children's difficulty with angles is not a consequence of a word-learning bias, per se, but rather a more general inability to separate angle from other dimensions, they should have difficulty taking advantage of training in either condition.

We chose to study this question in preschoolers for a number of reasons. First, children at this age are unlikely to have received previous instruction on angles, allowing us to experimentally manipulate children's exposure to the word "angle" without confounds that may exist based on prior knowledge. We also reasoned that this would provide the strongest test of our hypothesis since younger children are generally more prone to combining separable dimensions into a single representation (Kemler & Smith, 1978). Thus, if children at this age, who are given input that helps them overcome the whole-object bias, are able to understand angle better than those who are not given this kind of support, our findings would suggest that the errors of older students are unlikely to be due solely to a representational limit or inability to separate angle measure from other dimensions. Accordingly, the present study had two goals. Our first goal was to discover if the angle misconception that is exhibited by older students is also present in preschool-aged children. If so, our second goal was to test whether this misconception could be overcome by providing children with clearer spatial language, namely, a separate label for the overall angle figure. Such a result would suggest that children are able to attend to angle as an independent dimension of the overall figure, but are sometimes misled by ambiguous language.

Method

Participants

Thirty 4- to 5-year-old children ($M = 4.86$, $SD = .53$) participated in the study ($n = 15$ male). Children were recruited through a private urban preschool and were tested if their parents returned a signed consent form, which had been sent home with children along with information about the study. Participation was voluntary and no compensation or gift was given for participation.

Design and Procedure

In a single session, children received a pretest, a brief training session consisting of three parts (introduction, description, and guided practice), and a posttest. All children received the same 6 pretest and 16 posttest trials, which were presented without feedback. For training, children were randomly assigned to either an experimental or a control group. While the exact same stimuli were used for the experimental and control training, the conditions differed slightly in terms of the experimenters' descriptions and questions associated with the stimuli during training. In the experimental condition, children only heard the word "angle" in reference to the measure of rotation of an angle figure and were given a separate label, "toma" to represent the overall angle figure. By contrast, the control condition was designed to mimic the verbal input that is typically received in a classroom setting. Thus, children in this condition never heard a separate label for the overall figure ("toma"), which according to our hypothesis could make the referent of "angle" ambiguous. More specifically, we reasoned that if children in the experimental condition outperformed those in the business-as-usual control condition on posttest, we could safely conclude that the additional "toma" whole-object label helped children deduce the proper referent of "angle" over and above giving children a classroom-like lesson on angles with practice and feedback on angle comparison questions.

Pretest and Posttest Task

On each pretest and posttest trial, all children were presented with a card depicting two angle figures and asked: "Can you show me the bigger angle?" Each angle figure was formed by two line segments that met at a single point (to form an angle). All figures were arranged in the same orientation with a horizontal base and the vertex on the left side of the page.

The length of the line segments and measure of the angles varied such that there were three trial types. On *length-consistent* trials, the figure with the larger angle also had longer sides than the figure with the smaller angle; on *length-neutral* trials the figures with the larger and smaller angle had sides of equal length; and on *length-inconsistent* trials, the figure with the larger angle had shorter sides than the figure with the smaller angle (Figure 1).

A response was marked correct if a child pointed to the angle of greater measure. Therefore, on length-consistent trials, participants could respond correctly even if they based their selections on the length of the angle figures' sides. On length-neutral trials this cue was removed but children could still succeed by relying on other cues such as the distance between the endpoints of the sides. Consequently, our primary measure was performance on length-inconsistent trials since children should fail on these trials if they rely on cues other than rotational measure.

Training: Part I

In the introduction phase of training, all children saw a picture of a single acute angle figure. For children in the experimental condition, the experimenter pointed and said: "This is a toma. Can you say toma?" For children in the control condition, the experimenter pointed and said: "This is an angle. Can you say angle?" Next, all participants saw a series of four pairs of angle figures (two pairs of acute angles and two pairs of obtuse angles). The angle figures within each pair differed in side length but did not differ in angle measure, distinguishing them from the three pretest and posttest trial types. In the experimental condition, the experimenter pointed to each figure and said: "Here are two tomas. Here is a bigger toma and here is a smaller toma. Can you point to the bigger toma? Can you point to the smaller toma?" Each of the four trials was repeated and the experimenter provided feedback regardless of whether the child was correct or incorrect (i.e., "Right! This is the bigger toma!" or "Oops! This is the bigger toma"). In the control condition, children saw the same four pairs of angles, but rather than drawing the child's attention to the overall size of the figure with the label "toma," the experimenter simply pointed to each figure and said "Here are two angles. Here is an angle and here is an angle. Can you point to an angle? Can you point to the other angle?" Each of the four trials was repeated and the experimenter provided encouragement each time the child correctly pointed to the two angles.

Training: Part II

In the description phase of training, all participants were shown a picture of a single angle figure with the arc of the angle highlighted by an arrow. In the experimental condition, participants were told:

> Let's take a closer look at the toma. There are two lines [experimenter traces sides] the top line opens up [experimenter traces arrow] to form an angle [experimenter points to the center of the figure].

This was repeated three times for each child.

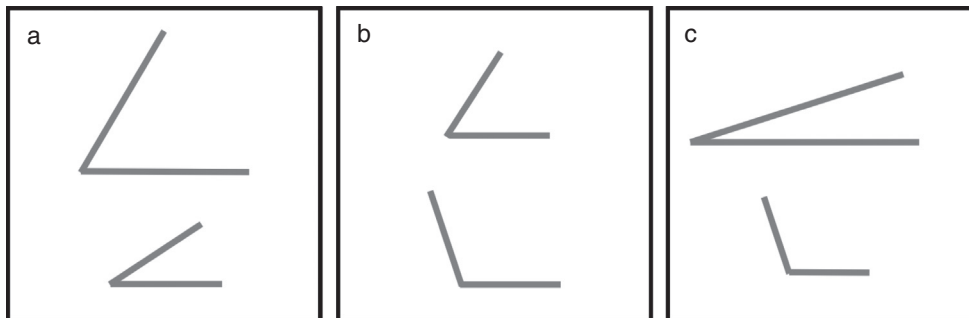In the control condition, the participants were told:

Figure 1. Sample stimuli for each trial type: length-consistent (a), length-neutral (b), and length-inconsistent (c).

Let's take a closer look at the angle. There are two lines [experimenter traces sides] the top line opens up [experimenter traces arrow] to form an angle [experimenter points to the center of the figure].

Again, this was repeated three times for each participant.

Training: Part III

During the final training phase, the guided practice phase, participants were given three length-consistent and three length-inconsistent trials that were presented in a fixed, random order. In the experimental condition, children were presented with the first pair of angles and were asked: "Can you point to the bigger toma?" They were given feedback ("Right! This is the bigger toma" or "Oops! This is the bigger toma") and then told:

Now let's look at the angles. This is the bigger angle [experimenter points to center of the figure with the larger angle] and this is the smaller angle [experimenter points to center of other figure]. Can you show me the bigger angle?

Again, children received feedback ("Right! This is the bigger angle" or "Oops! This is the bigger angle"). This process was repeated for all six trials. After going through all trials once, the same six trials were repeated except during the second time through, participants were only asked, "Can you show me the bigger angle?" with no mention of the "toma." Again, all participants received feedback on these training practice trials regardless of whether or not they were correct.

In the control condition, children saw the same six trials, but were not asked to identify the larger toma. Instead they were told: "Here are two angles. This is the bigger angle [experimenter points to center of one figure] and this is the smaller angle [experimenter points to center of other figure]. Can you show me the bigger angle?" The six trials were repeated a second time, and participants were only asked "Can you show me the bigger angle?" All participants received feedback on these practice trials regardless of whether or not they were correct as was the case in the experimental condition.

Results

A 2 (condition: experimental, control) × 3 (type: length-consistent, length-neutral, length-inconsis-

tent) × 2 (session: pretest, posttest) mixed-design analysis of variance (ANOVA) on children's accuracy revealed a main effect of session, $F(1, 168) = 7.66$, $p = .006$, reflecting children's better performance at posttest ($M = 78.6\%$ correct) than pretest ($M = 66.4\%$ correct). There was also a main effect of type, $F(2, 168) = 94.70$, $p < .001$, that was qualified by a Session × Type interaction, $F(2, 168) = 6.96$, $p = .001$. Finally, there was a main effect of condition, $F(1, 168) = 11.80$, $p < .001$, that was qualified by a Session × Condition interaction, $F(1, 168) = 5.07$, $p = .026$.

To further explore the interaction terms, we subsequently ran two separate ANOVA models, one for pretest and one for posttest. At pretest, our 2 (condition: control, experimental) × 3 (type: length-consistent, length-neutral, length-inconsistent) ANOVA on pretest scores revealed a main effect of type, $F(2, 84) = 61.47$, $p < .001$, but no effect of condition, $F(1, 84) = 0.57$, $p = .45$, as would be expected based on random assignment to the conditions (Figure 2). Participants in both conditions scored above chance (50%) on length-consistent—experimental, $t(14) = 7.48$, $p < .001$, and control, $t(14) = 7.87$, $p < .001$—and length-neutral—experimental, $t(14) = 4.78$, $p < .001$, and control, $t(14) = 4.13$, $p < .001$—trials, but below chance on length-inconsistent trials—experimental, $t(14) = -3.76$, $p < .001$, and control, $t(14) = -4.18$, $p < .001$. Thus, children did not select the correct angle when rotational measure was the only reliable cue.

At posttest, our 2 (condition: control, experimental) × 3 (type: length-consistent, length-neutral, length-inconsistent) ANOVA revealed a main effect of condition such that participants in the experimental condition outperformed those in the control condition, $F(1, 84) = 21.35$, $p < .001$ (Figure 3). Again, there was a main effect of type, $F(2$,
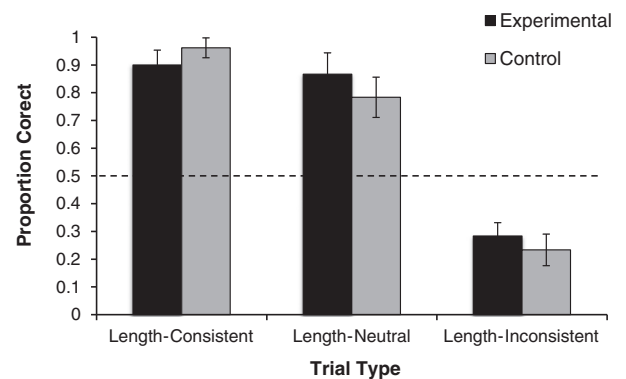

Figure 2. Average proportion of pretest trials correct by condition and trial type.
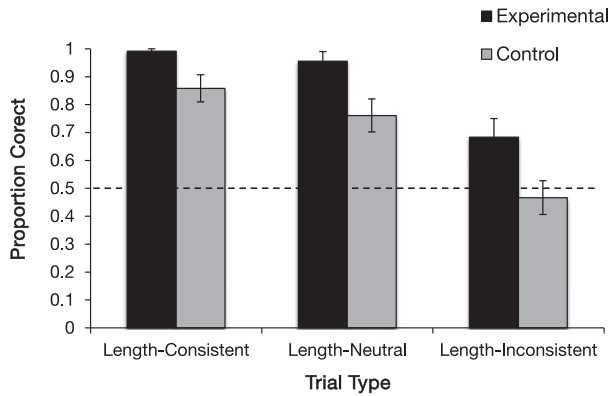
Figure 3. Average proportion of posttest trials correct by condition and trial type.

84) = 33.34, $p < .001$; performance was lower on length-inconsistent trials than on length-consistent, $t(58) = 7.19$, $p < .001$, and length-neutral, $t(58) = 5.26$, $p < .001$, trials. Participants in both conditions scored above chance on length-consistent—experimental, $t(14) = 31.60$, $p < .001$, and control, $t(14) = 6.33$, $p < .001$—and length-neutral—experimental, $t(14) = 12.24$, $p < .001$, and control, $t(14) = 4.04$, $p < .001$—trials. Importantly, participants in the experimental condition also performed significantly above chance on length-inconsistent trials, $t(14) = 2.17$, $p < .05$, whereas participants in the control condition did not perform differently from chance on this trial type, $t(14) = -0.72$, $p = .49$.

The distribution of individual participants' scores was consistent with these findings. Notably, the modal score of participants in the experimental condition on length-inconsistent trials was 75% correct (7 of 15 participants) with an additional 4 participants scoring above 75% correct. In contrast, the modal score of participants in the control condition on length-inconsistent trials was 50% correct (6 of 15 participants) with only 2 participants scoring 75% or better (Figure 4).
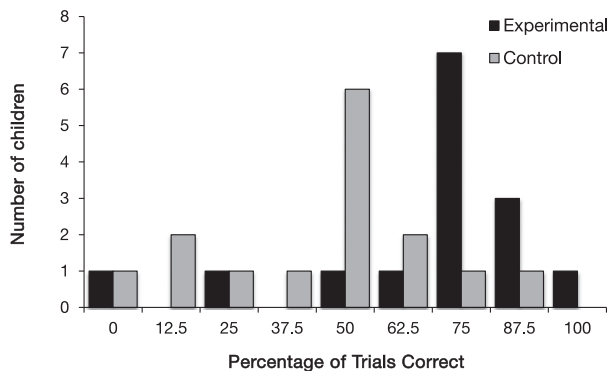

Figure 4. Distribution of participants' scores on length-inconsistent trials by condition.

Discussion

The present study clarifies the seemingly conflicting literature on children's understanding of angle and, more broadly, sheds light on the role of word-learning principles in guiding conceptual development. Within the context of angle, our results first demonstrate that under the right conditions, even preschool children are capable of learning about angles. Second, we show that acquiring a correct understanding of "angle" is largely a reference problem and can be influenced by children's own word-learning biases combined with the language used to describe angles.

Our study design was predicated on the assumption that preschool children in our sample would show the same angle misconception and error pattern as has been reported in older students. To confirm this, we administered a pretest prior to any training. As expected, whether angle measure positively correlated with side length (length-consistent trials) or negatively correlated with side length (length-inconsistent trials), participants showed a preference for selecting the figure with longer sides. The generally high performance on length-neutral trials could be interpreted in one of two ways. First, it is possible that holding side length constant causes children to correctly attend to and make their selections using angle measure. However, on these length-neutral trials, the figure with the greater angle measure *also* appeared to be a larger overall figure (greater in height and/or width) than the figure with the smaller angle. Therefore, an alternative explanation of participants' above-chance performance on length-neutral trials is that children base their selections on overall size of the figure rather than on angle measure. This possibility is most consistent with the overall pattern of responses on the other two trial types as well as with our hypothesis that children initially believe "angle" is a label for the whole figure and therefore mistakenly believe "bigger angle" refers to the larger figure.

After confirming the existence of the expected error, we directly tested whether the whole-object bias contributed to children's difficulty by giving participants in the experimental condition a separate label for the whole figure during training. We found that these participants were much more likely to benefit from the short lesson on angle and to select the larger angle on the critical length-inconsistent posttest trials. The fact that this improvement on length-inconsistent trials was accompanied by continued above-chance

performance on length-consistent and length-neutral trials suggests children did not merely flip their rule (i.e., select the angle figure with shorter sides). To succeed on all three trial types, children must have disregarded the side length of the angle figures and based their answers solely on angle measure rather than other competing cues to size.

We consider two interpretations regarding children's ability to perceive angle as a fully independent dimension following the experimental training. First, it is possible that children's prelinguistic understanding of angles is marked by the mistaken perception that increasing an angle figure's sides also results in a wider angle. If this were the case, our training could be seen as directing children's attention to the fact that angle measure varies independently of other dimensions, such as the size of the overall figure. We have reason to believe this interpretation is unlikely because the training session lasted only 5–10 min and involved only static images of angle figures. If this interpretation were correct, it would suggest that our brief, simple training session drastically changed children's perception, allowing them to perceive angles in a way that were not capable of before.

We believe a more parsimonious interpretation of the results is that children were already capable of perceiving angle as an independently varying dimension, but struggled to map the word "angle" onto that dimension without the help of a second label for the whole figure to disambiguate the referent. In other words, children's whole-object word-learning bias in combination with typical instructional language concealed their true capacity to grasp the important spatial concept of angle. This supports the work of researchers who have argued that very young children possess a robust ability to perceive and represent angle abstractly (e.g., Izard et al., 2014; Spelke et al., 2010). Moreover, the fact that 4-year-olds demonstrated this proficiency following the introduction of a word for the whole angle figure indicates that similar errors made by school-aged children are not likely due to conceptual limits on their representation of angle.

In either case, it is clear that leveraging children's mutual exclusivity bias allowed us to drastically improve children's understanding of "angle." One important question this raises is whether a second label is necessary for teaching young children about "angles" or if other methods could be similarly effective. While our results suggest that persistent misconceptions about angle stem largely from ambiguity in the referent of "angle," this ambiguity can likely be addressed through means other than the introduction of an additional whole figure label. Our findings should be regarded as diagnostic of the particular difficulty that children face when learning about angles and not necessarily prescriptive of the best instructional method. Importantly, our control condition was only designed to mimic typical classroom training while controlling for key features of the experimental training, in order to test our hypothesis. Even given this nonideal training, children in the control condition did make some improvements, for instance, from below-chance to chance-level performance on length-inconsistent trials.

Although other training methods might prove to be more practical in an educational context, our simple manipulation of children's mutual exclusivity bias not only proved to be highly effective but it also revealed an interesting fact about the source of children's misconceptions. While previous studies have found that the whole-object bias leads to difficulty learning certain types of words (e.g., collective nouns), such mapping errors are generally thought to be resolved before children reach school (Huntley-Fenner, 1995). Here, we have found evidence that the whole-object bias is at the root of a misconception that is prevalent well into children's elementary and even middle school education (Lehrer et al., 1998). This finding expands our understanding of how word-learning biases can have persistent effects on conceptual development.

Furthermore, we have demonstrated that certain misconceptions can be avoided by recognizing how children attach words to meanings and taking steps to ensure that their mappings are correct. While educators' intuitions may be to simplify instruction by introducing one concept at a time, our findings suggest that children benefit from the opportunity to contrast easily confusable meanings. It is likely that there are many more cases in which apparent misconceptions are the result of simple word-learning difficulties. For instance, children commonly confuse area with the length of a single side or the perimeter of a shape (Silverman, York, & Zuidema, 1984). By introducing these concepts together, children may gain a more accurate understanding of each concept, an idea that is supported by much research on analogy and structural alignment as a powerful learning tool (e.g., Gentner & Markman, 1994, 1997).

In sum, the present study is a striking example of how a serious misconception can arise from a disconnect between children's intuitions and the input they receive. Accordingly, our findings demonstrate a unique way to take advantage of

children's own mutual exclusivity bias to both test and resolve such cases. These results therefore affirm the power of language and children's word-learning biases to either cause basic misconceptions or to promote conceptual development.

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