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Approximate number word knowledge before the cardinal principle

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ABSTRACT

Approximate number word knowledge—understanding the relation between the count words and the approximate magnitudes of sets—is a critical piece of knowledge that predicts later math achievement. However, researchers disagree about when children first show evidence of approximate number word knowledge—before, or only after, they have learned the cardinal principle. In two studies, children who had not yet learned the cardinal principle (subset-knowers) produced sets in response to number words (verbal comprehension task) and produced number words in response to set sizes (verbal production task). As evidence of approximate number word knowledge, we examined whether children's numerical responses increased with increasing numerosity of the stimulus. In Study 1, subset-knowers (ages 3.0–4.2 years) showed approximate number word knowledge above their knower-level on both tasks, but this effect did not extend to numbers above 4. In Study 2, we collected data from a broader age range of subset-knowers (ages 3.1–5.6 years). In this sample, children showed approximate number word knowledge on the verbal production task even when only examining set sizes above 4. Across studies, children's age predicted approximate number word knowledge (above 4) on the verbal production task when controlling for their knower-level, study (1 or 2), and parents' education, none of which predicted approximation ability. Thus, children *can* develop approximate knowledge of number words up to 10 before learning the cardinal principle. Furthermore, approximate

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number word knowledge increases with age and might not be closely related to the development of exact number word knowledge.

Introduction

Approximate number word knowledge—understanding the approximate numerical magnitudes represented by number words or symbols—has been increasingly recognized as an important aspect of mathematical development (e.g., Booth & Siegler, 2008; Bugden & Ansari, 2011; Davidson, Eng, & Barner, 2012). Approximate number word knowledge is thought to reflect a mapping between number words and the approximate number system (ANS), which represents numerical quantity in a noisy fashion, with increasing variability as the quantity represented increases (e.g., Cordes & Brannon, 2008). Children's performance on tasks tapping approximate number word knowledge, such as symbolic number line estimation, speeded comparison of Arabic numerals, and labeling set sizes without counting, correlates with a variety of important mathematical outcomes, including standardized math achievement tests (e.g., Booth & Siegler, 2008; Bugden & Ansari, 2011; Davidson et al., 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013).

Despite the importance of approximate number word knowledge in children's mathematical learning, relatively little is known about the earliest stages of its development. Specifically, researchers disagree about *when* children map number words to the ANS; some claim that approximate number word knowledge begins to develop only after children learn the cardinal principle (that the last number reached when counting a set represents the whole set; Le Corre & Carey, 2007), and others claim that approximate number word knowledge begins to develop before children learn the cardinal principle (Wagner & Johnson, 2011).

The timing of the development of approximate number word knowledge has important theoretical ramifications. If children develop approximate number word knowledge only *after* learning the cardinal principle, then approximate number word knowledge *cannot* influence cardinal principle knowledge. In fact, some have argued that understanding the cardinal principle may actually be a necessary precursor to approximate number word knowledge (Le Corre & Carey, 2007). In contrast, if children acquire approximate number word knowledge *before* cardinal principle knowledge, then approximate number word knowledge may have a positive impact on the development of cardinal principle knowledge (Wagner & Johnson, 2011). In addition to these two conflicting causal accounts, we propose a third theoretical model in which exact number word knowledge and approximate number word knowledge develop separately and are not causally related. The goal of the current studies was to clarify the seemingly inconsistent extant findings and to provide insight into the developmental trajectory of children's approximate number word knowledge in relation to their cardinal principle knowledge.

Developmental trajectory of exact number word knowledge

A large body of research has charted the trajectory of how children learn exact cardinal number word meanings (e.g., understanding that the word "three" refers to sets of three entities) and the cardinal principle (for a review, see Carey, 2009). Children learn the cardinal meanings of the first three or four number words slowly and in order; during these stages, they are referred to as "one-knowers," "two-knowers," and so forth (Wynn, 1992). One-knowers know that the word "one" refers to sets of one item, and will use it correctly, but seem to use all other number words to mean merely "more than one" (Wynn, 1992). Several months later, they become "two-knowers," at which point they have learned that "two" refers to sets of two items but seem to use all other number words to mean "more than two." They then go through the same stages for "three" and sometimes "four" (Sarnecka & Lee, 2009; Wynn, 1992). Children in these stages are collectively referred to as "subset-knowers" because they understand the exact cardinal meanings of only a subset of the numbers in their count list. Although subset-knowers do not understand the exact cardinal meanings of the number words above their knower-level, they do know that these numbers differ from their known numbers, differ from each other, and are larger than their known numbers (Condry & Spelke, 2008; Wynn, 1992). Nevertheless, subset-knowers' knowledge of number words above their knower-level has generally been described as quite limited (e.g., Le Corre & Carey, 2007; Sarnecka & Lee, 2009; Wynn, 1992).

Carey (2009) developed a prominent theory, referred to as "bootstrapping," to explain the cognitive processes underlying this developmental trajectory. The bootstrapping theory posits that children learn the cardinal meanings of the first three (or four) number words by mapping these words onto the "enriched parallel-individuation system," which represents small numbers of items exactly (Le Corre & Carey, 2007). The enriched parallel-individuation system combines the representations formed through the parallel-individuation system (which operates through pointers that identify and track up to three or four individual entities at a time) with those of set-based quantification (which forms the basis for linguistic singular/plural distinctions) (e.g., Feigenson, Dehaene, & Spelke, 2004; Starkey & Cooper, 1980; Strauss & Curtis, 1981). After learning the exact cardinal meanings of the first three (or four) number words, children are unable to continue learning number words one at a time because higher set sizes (beyond four) cannot be represented by the enriched parallel-individuation system. Instead, children must learn a rule about how counting relates to set size, namely, the cardinal principle (Gelman & Gallistel, 1978).

Learning the cardinal principle is a major achievement in preschoolers' mathematical development. It typically takes 12 to 18 months from the time children learn the meaning of "one" to the time they understand this principle (Wynn, 1992). Once children understand the cardinal principle, they can represent the exact set sizes for all numbers in their count list, a prerequisite for succeeding on many other mathematical tasks such as matching highly dissimilar sets based on their set sizes and understanding that adding one to a set increases its number by exactly one (Davidson et al., 2012; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006; Mix, 2008; Wynn, 1990, 1992).

Developmental trajectory of approximate number word knowledge

Much of the work on adults' and children's approximate number word knowledge suggests that it takes the form of a "mapping" between number words and the ANS, at least among children who have already learned the cardinal principle (e.g., Huntley-Fenner, 2001; Le Corre & Carey, 2007; Lipton & Spelke, 2005; Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977; Sullivan & Barner, 2014). The ANS represents non-symbolic numerical quantity in a noisy fashion (for reviews, see Cordes & Brannon, 2008; Feigenson et al., 2004). Researchers have reported that children's estimates of large approximate numbers show scalar variability, a signature of the ANS (Huntley-Fenner, 2001; Le Corre & Carey, 2007; Wagner & Johnson, 2011). Scalar variability refers to a response pattern in which the standard deviation of responses divided by the mean response (also known as the coefficient of variation or COV) remains constant regardless of set size (e.g., Cordes, Gelman, Gallistel, & Whalen, 2001). For example, Huntley-Fenner (2001) showed 5- to 7-year-olds sets of 5 to 11 items and asked them to indicate the number of items. Children's responses and the standard deviations of their responses both increased with increasing set size. Importantly, their COV remained constant across set sizes, consistent with an underlying representation in which number words are mapped to the ANS.

Relation between exact number word knowledge and approximate number word knowledge

Le Corre and Carey (2007) examined young children's (ages 3–5 years) responses on a numerical approximation task and compared them with children's stage of exact cardinal number word knowledge. Subset-knowers produced similar number word responses regardless of whether they were asked to label 5 or 10 objects, suggesting that they did not yet map between approximate set sizes and number words. In the same study, some cardinal-principle-knowers said larger number words when shown larger set sizes (referred to as "mappers"), whereas others did not ("non-mappers")

(Le Corre & Carey, 2007). This study found that non-mappers tended to be younger than mappers. The researchers concluded that children acquire approximate number word knowledge (above their knower-level) only *after* learning the cardinal principle. These findings provided strong support for Carey's (2009) bootstrapping theory, in which children learn exact meanings for the first four number words by mapping them to states of the enriched parallel-individuation system, whereas approximate number word knowledge plays no role in children's learning of the cardinal principle. In other words, if children do not acquire approximate number word knowledge before learning the cardinal principle, as indicated by Le Corre and Carey's (2007) findings, then it is impossible for verbal approximation to facilitate cardinal principle knowledge.

However, a subsequent study suggested that children may possess approximate number word knowledge above their knower-level even before learning the cardinal principle (Wagner & Johnson, 2011). In this study, subset-knowers were asked to produce sets of objects of varying quantities (e.g., "Can you put six fish in the pond?"). They gave larger sets of objects when a larger number was requested, even above their knower-level, showing evidence of approximate number word knowledge. Furthermore, children's pattern of responses and errors showed evidence of scalar variability, suggesting (in contrast to Le Corre & Carey, 2007) that even subset-knowers were able to map number words to the ANS. Wagner and Johnson (2011) interpreted this finding as evidence that mapping number words to the ANS may facilitate children's acquisition of the cardinal principle.

We propose a third alternative—that the development of exact number word knowledge and the development of approximate number word knowledge are not causally related. Evidence for this lack of a causal relation comes from homesigners—individuals who are deaf, have not been exposed to a sign language, and communicate using gesture systems of their own creation. Adult homesigners use numerical gestures to approximate large set sizes but do not understand the cardinal principle (i.e., they cannot exactly enumerate sets of more than four items; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011). Thus, cardinal principle knowledge cannot be a necessary precursor to numerical approximation, and at the same time numerical approximation does not always lead to cardinal principle knowledge. Exact number word knowledge, including learning the cardinal principle, appears to rely heavily on linguistic input, especially number talk about present objects (e.g., Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Spaepen et al., 2011), whereas approximate number word knowledge may rely on age-related increases in other cognitive capacities, including ANS acuity, associative mapping ability, executive function, spatial skills, and fluency with the count sequence (e.g., Davidson et al., 2012; Halberda & Feigenson, 2008; Sullivan & Barner, 2014; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014; Wagner & Johnson, 2011). If exact number word knowledge and approximate number word knowledge develop independently, with approximate knowledge more tightly linked with age, then we might find evidence of approximate number word knowledge among some subset-knowers but not others and find that approximate number word knowledge increases with age but is unrelated to exact cardinal number word knowledge.

Examination of previous conflicting findings

The two previous studies of subset-knowers' approximate number word knowledge differed in several ways that might explain their seemingly discrepant results. First, the studies used different types of tasks. Le Corre and Carey (2007) used a *verbal production* task (Fast Cards); children saw a picture of objects and were asked to produce a corresponding number word without counting. Wagner and Johnson (2011) used a *verbal comprehension* task (Give-*N*); children needed to create a set of objects equal in value to the quantity requested verbally by the experimenter. Wagner and Johnson suggested that subset-knowers in their study may have shown evidence of ANS mapping because a verbal comprehension task places lower verbal demands on children than a verbal production task.

Second, the studies differed significantly in how the data were analyzed. Le Corre and Carey (2007) focused their analyses on set sizes from 5 to 10. Because children cannot represent numbers above 4 using the enriched parallel-individuation system, this analysis provides a strong test of mapping between the ANS and number words. In contrast, Wagner and Johnson (2011) analyzed all responses between a child's knower-level and 10 (e.g., for a two-knower, set sizes 3–10). Recent work suggests that subset-knowers possess some knowledge of the number just above their knower-level ($N + 1$) (Barner & Bachrach, 2010), leaving open the possibility that Wagner and Johnson's (2011) findings were driven by children's knowledge of $N + 1$ and not by children's mapping of the numbers above their knower-level to the ANS.

Finally, because these two studies were conducted using different participants, individual differences in the participants' characteristics or abilities may have driven the different results. The current studies used a within-participants design to eliminate this possibility.

The current studies

In these studies, we examined the approximate number word knowledge of children who have not yet learned the cardinal principle, referred to as subset-knowers. Although this term was originally coined to refer to children who know the exact meanings of a subset of the numbers in their count list (e.g., one-knowers, two-knowers, etc.), children who have not learned the exact meanings of any of the numbers in their count list ("pre-knowers") have also been referred to as subset-knowers (Le Corre & Carey, 2007). For simplicity, we use the term subset-knowers here to collectively refer to pre-knowers, one-knowers, two-knowers, three-knowers, and four-knowers.

In the two studies reported here, subset-knowers completed two types of numerical approximation tasks: a verbal comprehension task (Give-*N*) and a verbal production task (Fast Cards in Study 1 and Fast Dots in Study 2). In both studies, children completed both types of tasks, allowing us to compare the tasks within participants without the possibility of individual differences between children explaining differences in performance across tasks. We also performed our analyses in both of the ways described previously (using all numbers above a child's knower-level and using only numbers above 4) to determine whether differences in data analysis choices led to the discrepant findings.

In Study 1, we first attempted to closely replicate the previous findings using a within-participants design. Specifically, we recruited children (ages 3.0–4.2 years) to a university-based child development laboratory, resulting in a sample of middle- to high-socioeconomic status (SES) children, quite similar to the middle-class 3- and 4-year-olds sampled in previous studies (Le Corre & Carey, 2007; Wagner & Johnson, 2011). All children completed the Give-*N* task (previously used by Wagner & Johnson, 2011) and the Fast Cards task (previously used by Le Corre & Carey, 2007). We expected to replicate the results of both previous studies in a within-participants design. That is, we expected to find that subset-knowers show evidence of mapping on the Give-*N* task for numbers above their knower-level (Wagner & Johnson, 2011) *and* that subset-knowers *do not* show evidence of mapping on the Fast Cards task for numbers above 4 (Le Corre & Carey, 2007). Next, we examined whether differences in the *tasks* or differences in the *analyses* drove the different results in previous studies. To do so, we examined children's mapping ability on the Give-*N* task for sets above 4, an analysis not reported by Wagner and Johnson (2011). If children show evidence of mapping on the Give-*N* task even when examining only numbers above 4, then this suggests that the Give-*N* task is simply easier than the Fast Cards task, perhaps due to its lower demands on verbal production. If children fail to show evidence of mapping on the Give-*N* task for numbers above 4, then this suggests that subset-knowers' mapping ability on this task may be driven mainly by knowledge of their next number ($N + 1$) rather than a broader ability to map between the ANS and larger number words.

In Study 2, we sought to test a larger sample of subset-knowers from a broader age range (ages 3.1–5.6 years) in order to examine the unique contributions of age and knower-level to mapping ability among subset-knowers. On the verbal production task, prior work found that only older cardinal-principle-knowers (CP-knowers) showed approximate number word knowledge (Le Corre & Carey, 2007); however, the age range of subset-knowers was relatively restricted (the oldest subset-knower was 4.4 years old). We reasoned that it was possible that older subset-knowers would also show evidence of approximate number word knowledge, just as older CP-knowers do. To find older children who had not yet learned the cardinal principle, we sampled mainly from low-SES preschools based on previous work showing that low-SES children have lower number knowledge, on average, than middle-SES children (e.g., Jordan, Huttenlocher, & Levine, 1994; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). If mapping ability is related to age over and above knower-level, then approximate number word knowledge may be somewhat independent from knowledge of exact number word meanings.

In Study 2, we also chose to use a more recent version of the verbal production task, Fast Dots (Davidson et al., 2012). The Fast Dots task is more carefully controlled than Fast Cards in terms of timing (stimuli were presented on a computer screen for exactly 1 s instead of being presented on paper by an experimenter for approximately 1 s). Furthermore, set size and surface area were controlled (surface area covaried with set size on half of the trials and was held constant on the other half rather than covarying perfectly with surface area on all trials). The Fast Dots task also differed from Fast Cards in terms of the set sizes presented (1–51 vs. 1–9) and the way the task was introduced (the experimenter modeled responses for "eight," "fifteen," "thirty," and "fifty" vs. modeling a response only for "one"). Although we did not predict that these differences between Fast Dots and Fast Cards would significantly affect children's performance, we were nevertheless open to this possibility. For example, children may perform better on the Fast Cards task because surface area always covaried with number, or they may perform better on the Fast Dots task because the experimenter modeled use of larger number words that children might otherwise have difficulty in producing.

Study 1

Method

Participants

A total of 47 children (25 boys and 22 girls) participated (mean age = 3.44 years, *SD* = 0.27, range = 2.99–4.18). Children came from families of middle to high SES. Parents reported an average annual family income of $74,012 (*SD* = $26,289), with a range from $15,000–$35,000 to more than $100,000 (*N* = 43, with 4 parents not reporting their family income). Parents' education was recorded based on the maximum level of education reported for either parent. This level of education averaged 16.4 years (*SD* = 1.75), where 16 years is equal to a 4-year college degree. The range of parents' education was from some college to a graduate degree. Some of the 47 children did not complete all tasks; children were included in all analyses for which they had completed the relevant tasks. Therefore, sample sizes are slightly reduced from the maximum sample (*N* = 47) for each set of analyses.

An additional 48 children were initially screened but were ineligible for the study because they scored above the four-knower level on Give-*N* (*n* = 33), failed to count to at least 6 (*n* = 7), failed to complete either approximation task (*n* = 5), or had unclassifiable knower-level data (*n* = 3).[1] An additional 4 children completed the study but were excluded because their slope of responses was more than 3 standard deviations from the mean on one or more measures.

Design

Children completed three tasks as part of a larger battery of math-related tasks lasting 30 to 60 min. Children were tested at a child development laboratory. The Counting task assessed children's knowledge of the verbal count list. The Give-*N* task assessed children's ability to map number words to set sizes on a verbal comprehension task and also yielded their "knower-level" (Wynn, 1990, 1992). The Fast Cards task assessed children's ability to map number words to set sizes when verbal production of number words was required.

All children completed the Counting task first, followed immediately by the Give-*N* task. The Fast Cards task was always given last but did not always occur immediately after the Give-*N* task due to the inclusion of other tasks in the task battery.

[1] Some children had incomplete knower-level data due to experimenter error or children's refusal to respond on certain trials of Give-*N* (Study 1: *n* = 6; Study 2: *n* = 18). In some cases, it was impossible to determine the child's knower-level and the child was considered ineligible for the study (these children were noted as "unclassifiable" in the main text; Study 1: *n* = 3; Study 2: *n* = 9). In other cases, the available Give-*N* data were consistent with a specific knower-level and the child was assigned that knower-level and retained in our analyses (Study 1: *n* = 3; Study 2: *n* = 9). However, to ensure the robustness of the effects reported below, we re-ran our analyses excluding *all* children whose knower-level data were incomplete. All results reported as statistically significant at $p < .05$ remained significant or marginally significant at $p \leqslant .06$, and results reported as non-significant ($p > .05$) remained so.

Measures

Counting. Children's count list was elicited in one of two ways. Children were asked to count to 10 or to count stickers (10 stickers in a single row). Children who did not count to 10 without objects were given the opportunity to count the stickers. If a child failed to preserve one-to-one correspondence while counting the stickers, the experimenter asked the child to count again while the experimenter pointed to the stickers.

Give-N. In the Give-*N* task, children were given a pile of 15 plastic fish and were asked to put a certain number of fish (1–6) in a clear plastic bowl (Wynn, 1990, 1992). Each time a child responded incorrectly, the experimenter provided the opportunity for the child to fix his or her answer by saying, "Let's check. Can you count the fish?" After the child finished counting the fish, the experimenter said, "But I asked for *N* fish! Can you put *N* fish in the pond?" The child's answer after the correction prompt was used to determine his or her knower-level. The child's answer before the correction prompt was used in all analyses of approximation.

The experimenter started by asking for 1 fish. Subsequent numbers of fish were requested using the titration method modeled after Wynn (1992). If a child succeeded in giving *N* fish, $N+1$ fish were requested; otherwise, $N-1$ fish were requested. The titration phase ended when children gave *N* correctly at least two of three times and gave $N+1$ incorrectly at least two of three times or when the child gave "six" correctly two times. A child's knower-level (*N*) was determined based on his or her responses to set sizes 1 to 6 using the criteria established by Wynn (1992). First, the child responded correctly two of three times when asked for *N* objects and responded incorrectly two of three times when asked for $N+1$ objects. Second, the child responded by giving *N* objects no more than half as often (percentage-wise) when asked for higher set sizes than when asked for *N* itself (i.e., children did not frequently misuse their known numbers). The child's knower-level was determined as the highest number for which he or she met these two criteria. Children who gave the set sizes 1, 2, 3, and 4 correctly (either one of one time or two of three times) and gave 5 or 6 correctly at least two of three times were considered five-knowers (rare) or CP-knowers and were ineligible for the study.

After completing the titration phase, the experimenter requested the set sizes 4 and 6 one time each if the child had not already received them during the titration phase and then requested the set size 9 one time.

Although a reader might question the validity of using the same task (Give-*N*) to both classify children's knower-levels and examine their approximation ability, we believe that the knower-level classification criteria lead to a conservative test of approximation ability. Because the knower-level classification requires the child to be incorrect two of three times on the next number above his or her knower-level ($N+1$), this should, if anything, decrease our chances of finding evidence of approximation above *N* (because *N*-knowers are, by definition, inaccurate on $N+1$). However, our data show a significant positive slope from $N+1$ to 9 despite this bias (see Tables 2 and 4 in Results for Studies 1 and 2, respectively), suggesting that even though children are not accurate at $N+1$, they still show some knowledge that $N+1$ is lower than other higher numbers.

Fast Cards. The Fast Cards task was closely modeled after Le Corre and Carey's (2007) Fast Cards task. Children were told that the task was a guessing game and that the experimenter would show them pictures really fast and they needed to guess what they saw. The materials for the Fast Cards task consisted of 18 8.5×11-inch white sheets of paper with color pictures of objects printed on them. The sheets of paper were encased in clear plastic sheet protectors and presented in a three-ring binder. Children were shown three blocks of 6 cards for a total of 18 cards. Each block depicted one type of object (basketballs, trees, or bananas), and each card within the block depicted a different number of those objects (1, 2, 3, 4, 6, or 9). Within each block, the objects depicted were identical in all respects (e.g., all pictures of basketballs were the same size, shape, and color). The set sizes were presented in a fixed pseudo-random order within each block. The experimenter showed the child each card for approximately 1 s and then asked, "What's on the card?" After the child responded (typically by saying "ball" or "a ball"), the experimenter said, "That's right, it's ONE ball." For the rest of the cards, if a child did not produce a number word, the experimenter attempted to elicit a number word using the prompts "What else can you tell me?", "So what's on the card?", and "Can you take a guess?" or by

Table 1
Study 1: Ages of children by knower-level in maximum sample ($N = 47$).

Knower-level	n	Age (years)	
		Mean (*SD*)	Range
Pre-knowers	9	3.3 (0.2)	3.0–3.5
One-knowers	5	3.3 (0.2)	3.1–3.6
Two-knowers	15	3.5 (0.3)	3.0–4.2
Three-knowers	10	3.5 (0.2)	3.3–3.8
Four-knowers	8	3.6 (0.3)	3.1–4.0
Total	47	3.4 (0.3)	3.0–4.2

referring back to the first card and saying, "Remember, this is ONE ball. So what's on this card?" The experimenter showed the card to the child a second time while attempting to elicit a response.

Results

Counting

We measured children's highest number counted with no errors. In some cases, children were too shy to complete the Counting task, which was administered first, but later counted during another task in the same session. Children were given credit for the highest count list they produced with no errors on any task. For 87% of participants the highest count occurred during the Counting task, whereas for the remaining 13% of participants it occurred during a different task. We adopted the criterion used by Le Corre and Carey (2007) and included all children who successfully counted to "six" or higher with no errors. We note that the majority of children (87%) successfully counted to at least "nine," the highest number assessed in our measures of approximation.

Knower-levels

The number of children per knower-level and their average age are reported in Table 1.[2] As noted previously, an additional 33 children who scored above the four-knower level were screened but were considered ineligible for the study.

Approximation on Give-N

We first analyzed approximation on the Give-*N* task, where prior work suggests that subset-knowers should reveal evidence of mapping for numbers above their knower-level (Wagner & Johnson, 2011). Children who did not have at least 1 trial of Give-*N* on each of the set sizes $N + 1$, 6, and 9 were excluded ($n = 8$), leaving 39 children in the sample for analysis. The 8 excluded children were 3 pre-knowers, 2 one-knowers, 2 two-knowers, and 1 four-knower. Any differences between excluded and included children in knower-level, age, gender, family income, and parents' education did not reach statistical significance (all $ps > .05$).

Previous research has established that subset-knowers rarely count to get an exact answer on this task because these children do not yet understand how counting represents set size (Le Corre et al., 2006). This was also the case in our sample; across all participants, there was only 1 trial (of 191 analyzed trials) on which a child counted correctly and put the correct number of fish in the bucket, indi-

[2] We note that previous studies on this topic, although not explicitly excluding them, have included few or no pre-knowers (Le Corre & Carey, 2007; Wagner & Johnson, 2011). We chose to include pre-knowers because we were interested in understanding the approximate number word knowledge of all children who had not yet learned the cardinal principle. However, to make our findings more comparable to previous work, we also re-ran all analyses in both Studies 1 and 2 while excluding pre-knowers. Overall, the pattern of results remained the same, such that all analyses reported as significant at $p < .05$ remained significant or marginally significant at $p \leqslant .06$ and results reported as non-significant ($p > .05$) remained so. The only exception was that, after excluding pre-knowers, the effect of age on being a mapper on the Fast Dots task was no longer significant in Study 2. However, the analyses across Studies 1 and 2 remained the same when pre-knowers were excluded, showing that age, and not knower-level, study, or parents' education, was a significant predictor of being a mapper on the verbal production tasks.

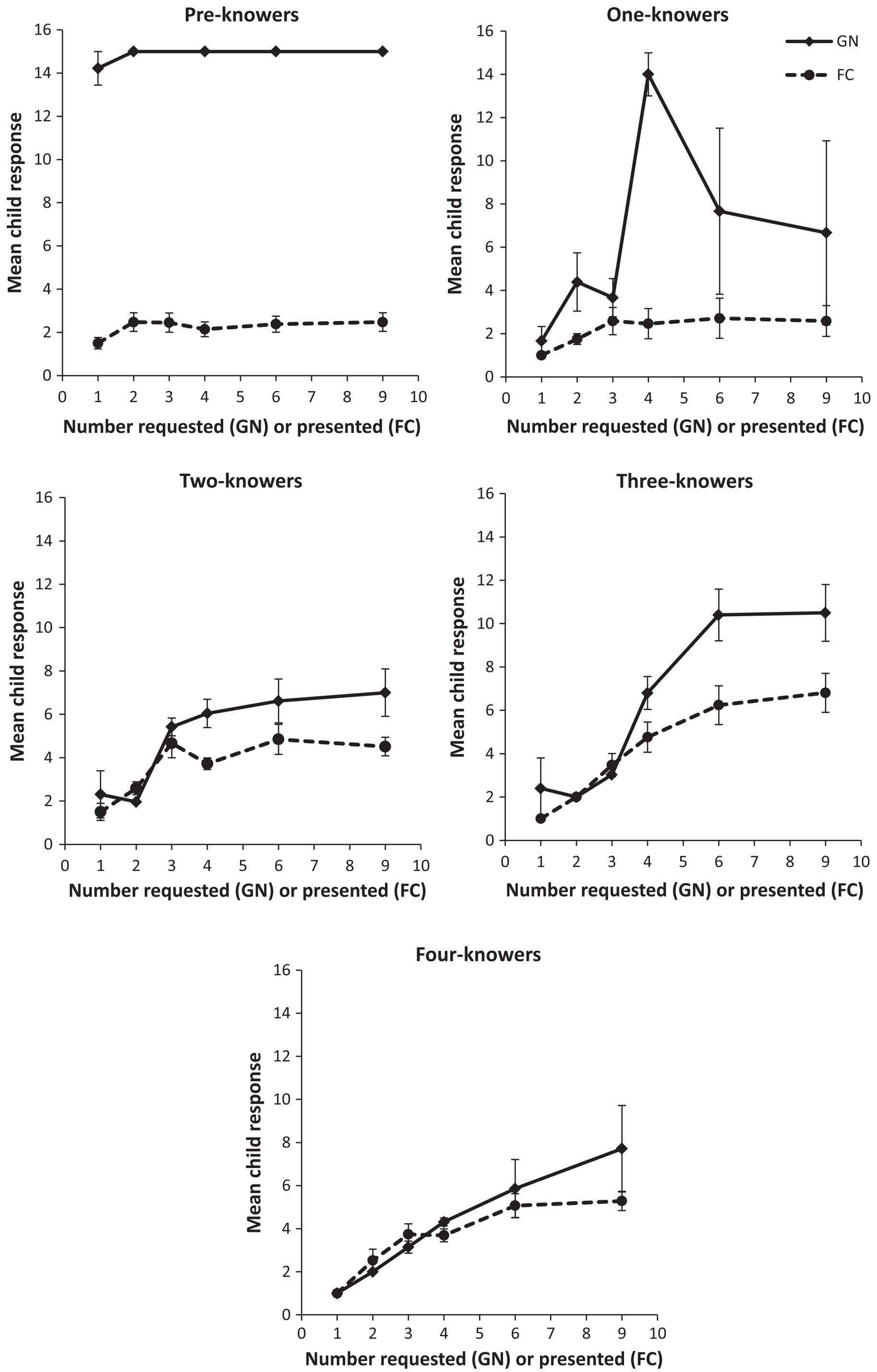

Fig. 1. Study 1: Average responses on Give-*N* (GN) and Fast Cards (FC) by set size and knower-level. Error bars represent 1 standard error.

Table 2
Study 1: Slopes of responses on Give-*N* and Fast Cards by knower-level.

Knower-level	Give-*N*			Fast Cards		
	Participants	*N* + 1 to 9 mean (*SD*)	6 to 9 mean (*SD*)	Participants	*N* + 1 to 9 mean (*SD*)	6 to 9 mean (*SD*)
Pre-knowers	6	.064 (.157)	.000[a]	7	.070 (.081)	.032 (.166)
One-knowers	3	.333 (.876)	−.333 (3.93)	4	.084 (.143)	−.042 (.285)
Two-knowers	13	.245 (.704)	.128 (1.68)	13	.046 (.186)	−.111 (.527)
Three-knowers	10	.684* (.884)	.033 (1.98)	7	.392** (.216)	.191 (.587)
Four-knowers	7	.619 (1.46)[b]	.619 (1.46)	7	.071 (.448)[b]	.071 (.448)
Total	39	.404** (.882)	.137 (1.74)	38	.123** (.266)	.012 (.449)

Note. Significance levels indicate difference from zero: *$p < .05$; **$p < .01$.
[a] The standard deviation could not be calculated because all slopes were equal to zero.
[b] The *N* + 1 to 9 slope for four-knowers was from 6 to 9 because 5 was not assessed on this task.

cating that children were not using counting to solve this task in an exact manner for numbers above their knower-level. This trial was excluded from our analyses.

Responses above a child's knower-level (N + 1 to 9). The average number of objects produced in response to each set size requested is displayed in Fig. 1. We first analyzed children's slope of responses above their knower-level (i.e., *N* + 1 to 9). We calculated "slope" by averaging each child's responses for each set size and then calculating the slope of the best-fitting regression line relating that child's average response by set size to the set sizes requested. The average slopes of children's responses from *N* + 1 to 9 are displayed in Table 2. Across all subset-knowers, the average slope of responses above a child's knower-level was significantly greater than zero ($M = .404$, $SD = .882$), $t(38) = 2.86$, $p < .01$, $d = 0.46$.

Responses above 4 (6–9). We next asked whether children demonstrated approximate number word knowledge for numbers that cannot be represented by the enriched parallel-individuation system (i.e., numbers greater than 4). To do so, we measured the slope of children's responses to the numbers 6 and 9 (Table 2). Across all subset-knowers, the average slope of responses from 6 to 9 was not significantly different from zero ($M = .137$, $SD = 1.74$), $t(38) = 0.49$, $p = .63$, $d = 0.08$. This null result suggests that the positive slope found when analyzing set sizes from *N* + 1 to 9 was driven by the numbers at or near *N* + 1. A follow-up analysis examining only set sizes from *N* + 2 to 9 (e.g., for a two-knower, set sizes 4 to 9) found that the average *N* + 2 to 9 slope was also not significantly different from zero ($M = .140$, $SD = 1.42$), $t(38) = 0.61$, $p = .54$, $d = 0.10$.

Approximation on Fast Cards

We next analyzed approximation on the Fast Cards task. On this task, previous research provides evidence of approximate number word knowledge for numbers above 4 among some cardinal-principle-knowers but not among subset-knowers (Le Corre & Carey, 2007). To remain consistent with previous research (Le Corre & Carey, 2007), we excluded trials on which children's cardinal response was greater than 30 and trials on which children refused to respond. Children who did not have at least 1 valid trial on each of the set sizes *N* + 1, 6, and 9 were excluded ($n = 9$), leaving 38 children for analysis. The 9 excluded children were 2 pre-knowers, 1 one-knower, 2 two-knowers, 3 three-knowers, and 1 four-knower. Any differences between the 9 excluded children and the 38 included children in terms of age, knower-level, gender, family income, or parents' education did not reach statistical significance (all $ps > .05$).

Responses above a child's knower-level (N + 1 to 9). Children's average responses to each set size, by knower-level, are displayed in Fig. 1, and their average response slopes are displayed in Table 2. Across all subset-knowers, the average slope of responses above a child's knower-level (*N* + 1 to 9) was significantly greater than zero ($M = .123$, $SD = .266$), $t(37) = 2.85$, $p < .01$, $d = 0.46$.

Responses above 4 (6–9). As we did on the Give-*N* task, we next asked whether children distinguished between set sizes outside of the subitizing range, 6 and 9. The slopes of children's responses on 6 to 9 are displayed in Table 2. The average 6 to 9 slope was not significantly different from zero ($M = .012$, $SD = .449$), $t(37) = 0.16$, $p = .87$, $d = 0.03$. As we did for the Give-*N* task, we examined whether the positive $N + 1$ to 9 slope on Fast Cards was driven by $N + 1$ by analyzing the $N + 2$ to 9 slope. Unlike on the Give-*N* task, the $N + 2$ to 9 slope was marginally different from zero and similar in magnitude to the $N + 1$ to 9 slope ($M = .098$, $SD = .324$), $t(37) = 1.86$, $p = .07$, $d = 0.30$, suggesting that the positive $N + 1$ to 9 slope may reflect knowledge of numbers from $N + 1$ to 4 rather than being solely driven by partial knowledge of $N + 1$.

Within-participants performance on Give-N and Fast Cards

Finally, we examined children's approximate number word knowledge across tasks. A total of 30 children completed both the Give-*N* and Fast Cards tasks. Using a within-participants pairwise comparison, these children's response slopes did not differ significantly between the Give-*N* task and the Fast Cards task either for the range of $N + 1$ to 9, $t(29) = 1.53$, $p = .14$, $d = 0.28$, or for the range of 6 to 9, $t(29) = 0.65$, $p = .52$, $d = 0.12$. We also asked whether children's performance was correlated between the two tasks. Children's slopes on the Give-*N* task were not significantly correlated with their slopes on the Fast Cards task either for the range of $N + 1$ to 9, $r(28) = .29$, $p = .12$, or for the range of 6 to 9, $r(28) = .17$, $p = .37$.

Discussion

In Study 1, subset-knowers had significantly positive response slopes on both the Give-*N* task (i.e., gave larger sets when asked for larger numbers) and the Fast Cards task (i.e., said larger numbers when asked for larger sets) for set sizes above their knower-level ($N + 1$ to 9). However, children did not have significantly positive response slopes on either task when only numbers above 4 were examined. These findings suggest that these subset-knowers had localized knowledge that the numbers from $N + 1$ to 4 are smaller than all other higher numbers. The results are in line with previous studies in finding a positive response slope on the Give-*N* task for sets above children's knower-level (Wagner & Johnson, 2011) and a flat response slope on the Fast Cards task for sets above 4 (Le Corre & Carey, 2007). Critically, we found that the previous discrepancies in the literature appeared to result from the range of sets analyzed more so than the task used given that the response slope from $N + 1$ to 9 was significantly positive, regardless of task, and the response slope above 4 (6–9) was not significantly different from zero regardless of task.

Study 1 replicated previous work by finding that subset-knowers did *not* show evidence of mapping number words to the ANS using a strict measure (i.e., only responses above 4). However, we wondered whether the narrow age range used in Study 1 and previous work (mainly 3-year-olds and young 4-year-olds) may have limited our ability to find subset-knowers capable of showing approximate number word knowledge for larger sets. In other words, if approximate number word knowledge (for sets above 4) improves with age, it is possible that children's young age, rather than their status as subset-knowers, limited this ability. In Study 2, we sought to more fully examine the relations among age, knower-level, and approximate number word knowledge. We recruited a larger sample of children from a broader age range (3.1–5.6 years). We targeted low-SES preschools because we wanted to find older children who had not yet learned the cardinal principle. Prior work in middle- to high-SES samples typically has not found subset-knowers older than 4.5 years (e.g., Le Corre & Carey, 2007; Sarnecka & Carey, 2008). Given that low-SES children have lower number knowledge, on average, than their middle- to high-SES peers (e.g., Klibanoff et al., 2006), we expected to find older subset-knowers in this population.

As noted in the Introduction, we made some modifications to the tasks in Study 2 in comparison with Study 1. The most significant change was that in Study 2 we used the Fast Dots task (Davidson et al., 2012) instead of the Fast Cards task (Le Corre & Carey, 2007) as our verbal production measure of approximate number word knowledge. We made this change because the Fast Dots task is more controlled in terms of presentation time (now computerized) and also controls for the non-numerical confound of surface area.

Study 2

Method

Participants

Participants in Study 2 were on average older and from lower socioeconomic backgrounds than participants in Study 1. A total of 79 children (47 boys and 32 girls) participated in Study 2 (mean age = 4.20 years, *SD* = 0.63, range = 3.11–5.57, *N* = 77; age was not available for 2 participants). Parents reported an average annual family income of $20,798 (*SD* = $23,313), with a range from less than $15,000 to more than $100,000 (*N* = 69; 10 parents did not report their family income). Parents' education (the maximum level of education reported for either parent) averaged 12.8 years (*SD* = 1.95, *N* = 71), where 12 years is equal to a high school degree. The range of parents' education was from less than high school to a graduate degree. As in Study 1, children were included in all analyses for which they had complete data; reductions in sample size from the maximum sample (*N* = 79) are explained in the Results.

An additional 156 children were screened but were ineligible for the study because they scored above the four-knower level on Give-*N* (*n* = 107), failed to count to at least 6 (*n* = 17), failed to complete either approximation task (*n* = 16), refused to speak or did not speak English (*n* = 5), had a developmental delay reported by the teacher (*n* = 2), or had unclassifiable knower-level data (*n* = 9). An additional 5 children completed the study but were excluded because their response slope was more than 3 standard deviations from the mean on one or more measures.

Design

Children were tested at their preschools. Similar to Study 1, children completed the Counting, Give-*N*, and Fast Dots tasks as part of a larger battery of math-related tasks lasting 30 to 60 min. The Give-*N* task was nearly identical to Study 1, whereas the Counting and Fast Dots tasks used in Study 2 differed from Study 1 in several ways explained below. As in Study 1, all children completed the Counting task first, then the Give-*N* task, and finally the Fast Dots task. The three tasks were usually separated by the inclusion of other number-related tasks in the task battery.

Measures

Counting. Because Study 2 was part of a larger study of numerical development, we chose to use a more sensitive measure of counting proficiency relevant to other research questions. Whereas in Study 1 children were asked to count to 10, in Study 2 children were asked to count as high as they could. If a child did not begin counting, the experimenter showed the child an array of 36 stickers (four rows of 9 stickers) and asked him or her to count the stickers starting with the top row. If the child stopped counting before reaching 10 (with or without stickers), the experimenter asked the child to start again. The highest number a child counted with no errors was recorded. For the current study, we were only interested in whether a child reached our minimum criterion by counting to "six" or higher with no errors.

Give-N. The Give-*N* task was administered the same way as in Study 1 with one exception. After the titration phase, the experimenter requested the set sizes 6 and 9 one time each if not already administered during the titration phase (in Study 1, the experimenter requested the set sizes 4, 6, and 9). We no longer required children to give "four" because 4 can be represented by the parallel-individuation system and so does not provide unique information about children's ability to map number words to the ANS. As in Study 1, children's responses after the correction prompts were used to determine their knower-levels, whereas their answers before the correction prompts were used in our analyses of approximation.

Fast Dots. We used the Fast Dots task developed by Davidson et al. (2012). As in the Fast Cards task, the Fast Dots task involved presenting sets of objects rapidly to prevent counting and asking children to label what they saw using number words. The Fast Dots task differed from the Fast Cards task in several ways. The Fast Dots task was administered on a computer with a standardized presentation time of 1 s per set of dots. The set sizes presented were 1, 2, 3, 5, 6, 8, 10, 12, 16, 20, 25, 30, and 51. The number of trials per set size ranged from 2 to 4, with 42 trials total. There were 5 introduction trials (purple dots) followed by 4 sets of dots that were introduced as different kinds of objects: green dots (“apples”), yellow dots (“stars”), blue dots (“raindrops”), and red dots (“tomatoes”). On half of the trials the size of the dots remained constant, whereas on the other half of the trials the total surface area remained constant. The trials were administered in a single pseudo-random order.

The experimenter introduced the task as a guessing game and modeled answers for the 5 introduction trials. For example, on the first introduction trial, 15 purple dots appeared for 1 s. The experimenter said, “Hmm, I think that was about fifteen dots.” The experimenter repeated this with subsequent trials of 50, 8, and 30 dots. The experimenter used words like “maybe” and “I think” to indicate uncertainty but always gave the correct answer. On the last introduction trial, the experimenter asked the child to label a set of 2 purple dots and then provided feedback. On subsequent test trials, the experimenter showed the dots for 1 s and then, after the dots disappeared, said, for example, “How many apples was that?” If a child looked away from the screen during the trial, the experimenter repeated that trial. If a child used a strategy for 3 trials in a row, such as counting (e.g., saying “six,” “seven,” and “eight” on subsequent trials) or repeating the same number, the experimenter asked the child to make sure he or she was taking really good guesses and to think about the answers. Children were given general encouragement during the test trials but were not given feedback about the accuracy of their responses.

Results

Counting

As in Study 1, we examined the highest count list a child produced with no errors on any task. For 85% of participants the highest count occurred on the Counting task, whereas for the remaining 15% of participants it occurred on a different task. All children who successfully counted to “six” or higher with no errors were included in our analyses. The majority of the included children (85%) successfully counted to at least “ten,” the highest number assessed in our measures of approximation.

Knower-levels

Knower-levels were determined using the same criteria as in Study 1. The number of children with each knower-level and their average age are reported in Table 3.

Approximation on Give-N

We first analyzed approximation on the Give-*N* task. Here, 2 children who did not have at least 1 trial on each of the set sizes $N + 1$, 6, and 9 were excluded (1 two-knower and 1 three-knower), leaving 77 children for analysis. Because only 2 children were excluded, we were not able to examine statistical differences in demographic characteristics and knower-levels between included and excluded

Table 3
Study 2: Ages of children by knower-level in maximum sample ($N = 79$).

Knower-level	*n*	Age (years)	
		Mean (*SD*)	Range
Pre-knowers	15[a]	4.3 (0.8)	3.3–5.5
One-knowers	8	3.9 (0.4)	3.4–4.4
Two-knowers	27	4.3 (0.7)	3.2–5.6
Three-knowers	18	4.1 (0.5)	3.1–4.9
Four-knowers	11	4.4 (0.7)	3.5–5.3
Total	79[a]	4.2 (0.6)	3.1–5.6

[a] Of the 15 pre-knowers, 2 did not have age data available.

children. As in Study 1, we excluded the few trials on which children counted correctly and put the correct number of fish in the bucket for sets above their knower-level (4 of 397 analyzed trials).

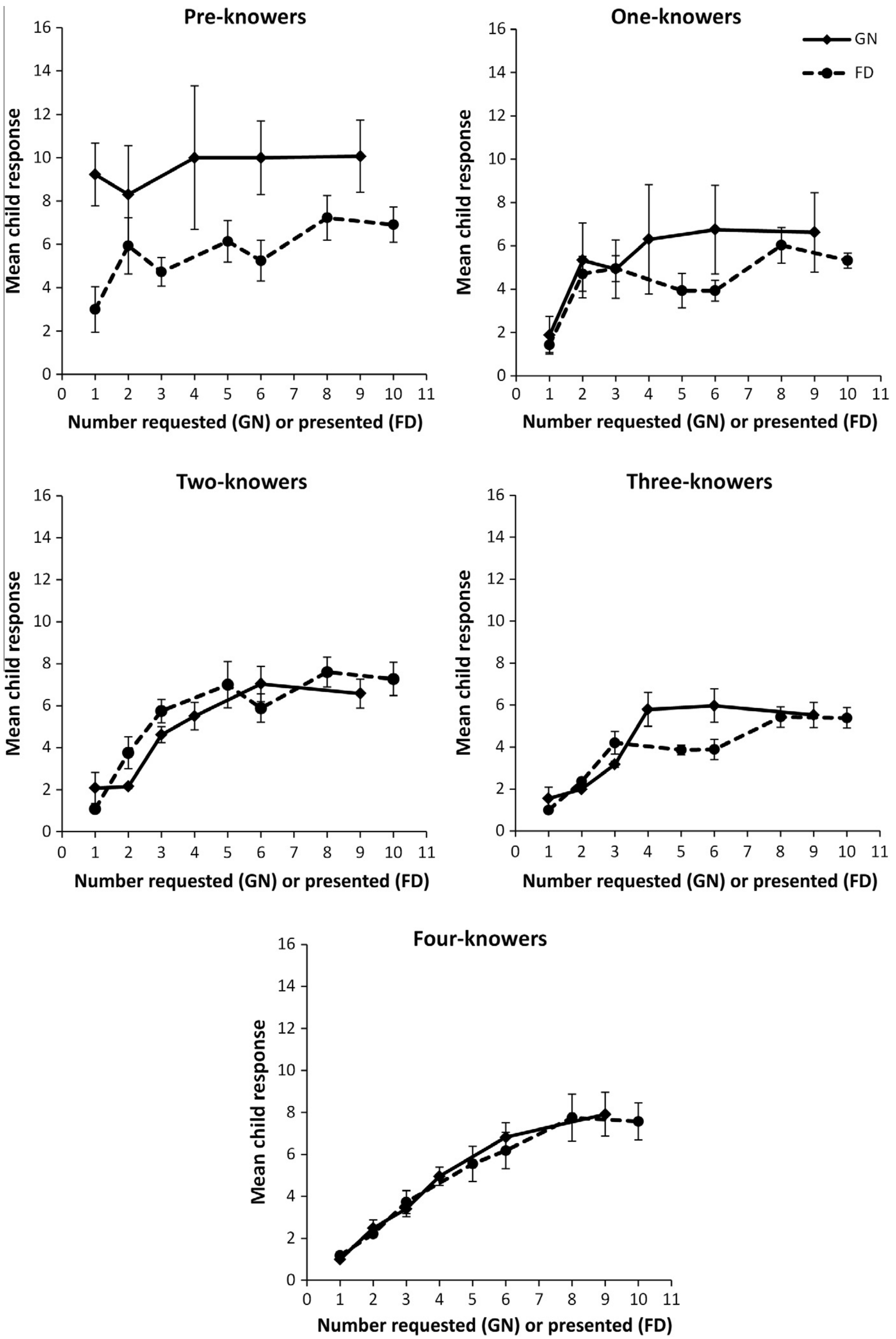

Fig. 2. Study 2: Average responses on Give-*N* (GN) and Fast Dots (FD) by set size and knower-level. Error bars represent 1 standard error.

Responses above a child's knower-level (N + 1 to 9). The average number of objects produced in response to each set size requested is displayed in Fig. 2. We calculated the slope relating children's responses to the number requested in the same way as in Study 1 (Table 4). Replicating Study 1, we found that the average slope of responses above a child's knower-level ($N + 1$ to 9) was significantly greater than zero ($M = .174$, $SD = .574$), $t(76) = 2.67$, $p < .01$, $d = 0.30$.

Responses above 4 (6–9). We next examined the slope of children's responses to the numbers 6 and 9 (Table 4). Replicating Study 1, the average slope of responses from 6 to 9 was not significantly different from zero ($M = -.032$, $SD = .878$), $t(76) = -0.32$, $p = .75$, $d = 0.04$. Also replicating Study 1, the $N + 2$ to 9 slope was not significantly different from zero ($M = .026$, $SD = .769$), $t(76) = 0.30$, $p = .77$, $d = 0.03$, suggesting that the positive $N + 1$ to 9 slope was driven mainly by children's partial knowledge of $N + 1$.

Approximation on Fast Dots

We next analyzed approximation on the Fast Dots task. As in Study 1, we excluded trials on which children's cardinal response was greater than 30 and trials on which children refused to respond. Children who did not have at least 1 valid trial of each of the set sizes $N + 1$, 5, and 10 were excluded ($n = 6$), leaving 73 children for analysis. The 6 excluded children were 4 pre-knowers and 2 two-knowers. The excluded children were significantly younger than the included children (excluded $M = 3.58$, $SD = 0.40$; included $M = 4.25$, $SD = 0.62$), $t(75) = -2.35$, $p < .05$, and were lower in knower-level than the included children (excluded $M = 0.67$, $SD = 1.03$; included $M = 2.14$, $SD = 1.25$), $t(77) = -2.80$, $p < .01$. Any differences between the 6 excluded children and the 73 included children in gender, family income, or parents' education did not reach statistical significance (all $ps > .05$).

Responses above a child's knower-level, up to 10 (N + 1 to 10). Children's average responses to each set size, by knower-level, are displayed in Fig. 2, and their average slopes are displayed in Table 4. On the Fast Dots task, the average slope of responses above a child's knower-level ($N + 1$ to 10) was significantly different from zero ($M = .300$, $SD = .270$), $t(72) = 9.49$, $p < .001$, $d = 1.11$.

Responses above 4, up to 10 (5–10). The slopes of children's responses above 4 (5–10) are displayed in Table 4. The average 5 to 10 slope was significantly different from zero ($M = .299$, $SD = .362$), $t(72) = 7.06$, $p < .001$, $d = 0.83$. Strikingly, and in contrast to the results of Study 1, subset-knowers were able to say larger number words for larger set sizes on this task even when those set sizes were above 4 and, thus, could not be represented by the enriched parallel-individuation system.

Responses above 10 (12–51). Children's average response slope for the range 12 to 51 (responses greater than 100 were truncated to 100 following Davidson et al., 2012) was not significantly different from zero ($M = .038$, $SD = .298$), $t(72) = 1.08$, $p = .29$, $d = 0.13$. Put another way, children's response to

Table 4
Study 2: Slopes of responses on Give-*N* and Fast Dots by knower-level.

Knower-level	Give-*N*			Fast Dots		
	Participants	*N* + 1 to 9 mean (*SD*)	6 to 9 mean (*SD*)	Participants	*N* + 1 to 10 mean (*SD*)	5 to 10 mean (*SD*)
Pre-knowers	15	.082 (.355)	.022 (1.05)	11	.361*** (.211)	.292* (.386)
One-knowers	8	.168 (.572)	−.042 (.547)	8	.118 (.160)	.365** (.285)
Two-knowers	26	.303** (.486)	−.154 (1.00)	25	.227*** (.225)	.176* (.406)
Three-knowers	17	−.060 (.642)	−.147 (.661)	18	.362*** (.277)[b]	.362*** (.277)
Four-knowers	11	.364 (.809)[a]	.364 (.809)	11	.435** (.374)	.435** (.374)
Total	77	.174** (.574)	−.032 (.878)	73	.300*** (.270)	.299*** (.362)

Note. Significance levels indicate difference from zero: $^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$.
[a] The $N + 1$ to 9 slope for four-knowers was from 6 to 9 because 5 was not assessed on this task.
[b] The $N + 1$ to 10 slope for three-knowers was from 5 to 10 because 4 was not assessed on this task.

12 dots ($M = 6.96$, $SD = 3.90$) did not differ significantly from their response to 51 dots ($M = 8.21$, $SD = 13.39$), $t(72) = 0.94$, $p = .35$, $d = 0.11$. To maintain consistency with previous work, we confine our further analyses to children's responses to numbers up to 10.

Knower-level and age as predictors of mapping ability. We next asked whether children's age or knower-level accounted for individual differences in mapping ability. Following the criterion used by Le Corre and Carey (2007), we categorized children as "mappers" if their response slope on the Fast Dots task for set sizes 5 to 10 was 0.3 or higher; all other children were considered "non-mappers." Of 73 subset-knowers, 32 children (44%) were categorized as mappers. In a logistic regression, knower-level did not significantly predict being a mapper ($\beta = 0.02$, $SE = 0.19$, $p = .91$). Indeed, mappers were similarly frequent among pre-knowers (45% of participants), one-knowers (63%), two-knowers (32%), three-knowers (44%), and four-knowers (55%).

In a separate logistic regression, age was a significant predictor of being a mapper ($\beta = 0.82$, $SE = 0.41$, $p < .05$). Mappers were on average 3.6 months older than non-mappers (mappers: $M = 4.41$ years, $SD = 0.56$; non-mappers: $M = 4.11$ years, $SD = 0.64$). In a logistic regression with age and knower-level entered simultaneously as predictors of being a mapper, age was a significant predictor ($\beta = 0.82$, $SE = 0.41$, $p < .05$), whereas knower-level was not ($\beta = -0.01$, $SE = 0.20$, $p = .96$).

Within-participants performance on Give-N and Fast Dots

A total of 71 children completed both the Give-*N* and Fast Dots tasks. For the range of $N + 1$ to 10, these children's response slopes were marginally higher on Fast Dots than on Give-*N*, $t(70) = 1.96$, $p = .054$, $d = 0.23$. For the range of 5 to 10, children's slopes were significantly higher on the Fast Dots task than on the Give-*N* task, $t(70) = 3.03$, $p < .01$, $d = 0.36$. As in Study 1, children's slopes were not significantly correlated between the Give-*N* and Fast Dots tasks either for the range of $N + 1$ to 10, $r(69) = .15$, $p = .20$, or for the range of 5 to 10, $r(69) = -.04$, $p = .72$.

Analyses comparing Fast Cards and Fast Dots

We next examined whether children's better approximation performance on Fast Dots (Study 2) compared to Fast Cards (Study 1) could be explained by differences in child age across the two populations or by differences between the tasks.

Age differences. To examine the effect of child age on the verbal production approximation tasks across Studies 1 and 2, we restricted our data to the age range that overlapped between both datasets, ages 3.1 to 4.2 years, resulting in a sample of 37 children from Study 1 and 35 children from Study 2. It was not possible to definitively disentangle the effects of age from study (Study 1 vs. Study 2) outside of this age range due to the lack of common support. We conducted a logistic regression predicting status as a mapper (5–10 slope > 0.30) on the Fast Cards or Fast Dots task, with age, knower-level, and an indicator for study (Study 1 = 0 and Study 2 = 1) entered simultaneously as predictors. Age was a significant predictor ($\beta = 2.93$, $SE = 1.12$, $p < .01$), whereas knower-level ($\beta = 0.06$, $SE = 0.23$, $p = .79$) and study ($\beta = -0.21$, $SE = 0.60$, $p = .73$) were not. In this subsample, the average age of non-mappers was 3.52 years ($SD = 0.29$, $n = 49$), whereas the average age of mappers was 3.75 years ($SD = 0.26$, $n = 23$). Thus, on average, mappers were 2.7 months older than non-mappers even within this restricted sample. Fig. 3 shows the percentages of children who were mappers by age group and study.

Because the samples in Studies 1 and 2 also differed in SES, we conducted a logistic regression including parents' education (as an indicator of SES) as well as child age, knower-level, and study. Parents' education was not a significant predictor of being a mapper ($\beta = 0.05$, $SE = 0.15$, $p = .74$). In this model, child age, but not knower-level or study, remained a significant predictor of being a mapper, suggesting that differences across the samples in child age, and not SES, best explained the results.

Task differences. Although the lack of a significant effect of study in the preceding analyses suggests that task differences are not the most plausible explanation for the differences between Study 1 and Study 2, we sought to verify this further by examining two specific differences between the tasks that may have affected the results. The first was that the range of set sizes measured was greater in the Fast Dots task (5–10) than in the Fast Cards task (6–9), which may have given children more of a

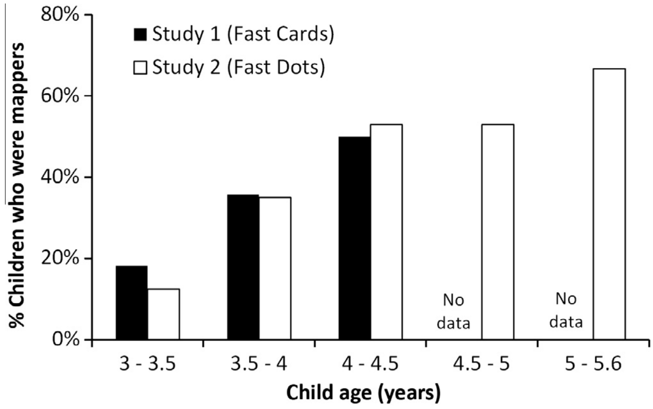

Fig. 3. Studies 1 and 2: Percentages of children, by age group and study, who were mappers for set sizes above 4 (slope >0.30) on a verbal production task. Note that Study 1 did not include children older than 4.5 years.

chance to show a positive slope on the Fast Dots task. Therefore, we examined children's Fast Dots slopes for the narrower ranges of 6 to 10 and 6 to 8. In both cases, we found that children still showed significantly positive slopes (6–10 slope: $M = .372$, $SD = .603$, $t(72) = 5.27$, $p < .001$, $d = 0.62$; 6–8 slope: $M = .894$, $SD = 1.39$, $t(71) = 5.47$, $p < .001$, $d = 0.64$).

The second potentially important difference between the tasks was that in the Fast Dots task the experimenter modeled the use of larger number words ("eight," "fifteen," "thirty," and "fifty") in the introduction trials, whereas in the Fast Cards task the experimenter modeled only the use of "one." It is possible that these trials primed children to use these words more often, thereby improving performance. If so, we would expect children to use the specific words that the experimenter modeled more frequently in the Fast Dots task than in the Fast Cards task. Although children did use the word "eight" more frequently (in response to set sizes 5–10) on the Fast Dots task than on the Fast Cards task, this was also true for the words "seven" and "nine" (all ps < .01). Furthermore, the "bump" in usage was not significantly greater for "eight" than for "seven" and "nine" (non-significant set size by study interaction, $F(1.8, 191.2) = 0.94$, $p = .39$), failing to support a specific priming effect. Use of "fifteen" and "thirty" was very rare and did not differ significantly across studies. We also re-ran our analyses of the Fast Dots task, excluding children's responses of "eight" and "fifteen," and the pattern of results remained unchanged. This suggests that experimenter modeling of larger number words is not likely to explain the differences in results across the two studies.

Discussion

For the Give-*N* task, Study 2 replicated the pattern of results from Study 1; subset-knowers showed positive mapping slopes above their knower-level, but these slopes became flat for sets above 4 and appeared to be driven by knowledge of $N + 1$. However, results for the verbal production task in Study 2 differed strikingly from Study 1. Subset-knowers showed positive mapping slopes on the Fast Dots task in Study 2 even for set sizes above 4. Post hoc analyses suggested that this was driven by the fact that children in Study 2 were older than those in Study 1. Within Study 2, mapping ability on the Fast Dots task was significantly related to age, controlling for knower-level, but was not related to knower-level. In addition, a comparison across Studies 1 and 2 showed that child age, and not knower-level or study, predicted whether children were mappers on the verbal production tasks. Alternative explanations involving task differences between Study 1 and Study 2 were not supported. Thus, older children were more likely to map number words to set sizes 5 to 10 on the verbal production approximation tasks regardless of their knower-level.

General discussion

The two studies reported here show that children *can* develop approximate number word knowledge for set sizes above their knower-level even before learning the cardinal principle. Most subset-knowers' approximate number word knowledge was limited to numbers just above their knower-level and did not extend to numbers above 4. Importantly, however, some subset-knowers showed approximate number word knowledge even for larger numbers (5–10). Subset-knowers' large-number approximation ability improved with age but not knower-level, suggesting that approximate number word knowledge and exact number word knowledge may develop through independent processes.

Study 1 replicated findings from previous work in a within-participants design; subset-knowers showed approximate number word knowledge above their knower-level (set sizes $N + 1$ to 9) on a verbal comprehension task (Give-*N*; Wagner & Johnson, 2011), but the same children failed to show approximate number word knowledge above 4 (set sizes 6–9) on a verbal production task (Fast Cards; Le Corre & Carey, 2007). Further analyses revealed that these seemingly conflicting results were driven by differences in the numerical ranges tested rather than the tasks themselves. On both tasks, children showed approximate number word knowledge when *all* numbers above their knower-level were examined ($N + 1$ to 9, a range not previously examined on the Fast Cards task) but not when only numbers above 4 were tested (a stricter test that was not previously applied to the Give-*N* task). This suggests that children's success on these tasks was driven by an understanding that the magnitudes of the number words just above their knower-level ($N + 1$ to 4) are less than those of higher numbers rather than by a more general approximation ability.

Study 2 aimed to disentangle the contributions of age and knower-level to approximate number word knowledge by examining subset-knowers from a wider range of ages (3.1–5.6 years), including older subset-knowers than have been assessed previously. Children's performance in Study 2 on the verbal comprehension task (Give-*N*) replicated their performance in Study 1; subset-knowers showed approximate number word knowledge for $N + 1$ to 9 but not 6 to 9. However, in contrast to Study 1 and previous work (Le Corre & Carey, 2007), in Study 2 children performed quite well on the verbal production task. Specifically, subset-knowers showed evidence of approximate number word knowledge (i.e., positive response slopes) on the verbal production Fast Dots task in Study 2 even when examining only numbers above 4, which cannot be represented by the parallel-individuation system. We categorized subset-knowers as mappers using the same criteria as in Le Corre and Carey (2007) and found that mappers were on average 3.6 months older than non-mappers. The relation between age and mapper status remained significant even after controlling for children's knower-levels, which were not significantly related to their status as mappers. Further analyses combining the data from Studies 1 and 2 showed that child age, and not knower-level, parent education, or task differences, best explained children's mapping ability on the verbal production tasks. Although Le Corre and Carey (2007) did not find evidence of mapping ability among subset-knowers using a verbal production task, we found such evidence among older but not younger subset-knowers. This is similar to Le Corre and Carey's (2007) finding that older cardinal-principle-knowers were more likely to be mappers than younger cardinal-principle-knowers. The relation between child age and mapping ability among both subset-knowers and CP-knowers suggests that the ability to map number words to the ANS develops with age and is somewhat independent of children's learning of the exact cardinal meanings of the number words (i.e., knower-levels) and the cardinal principle.

Our results have important implications for the possible causal relations between the development of exact number word knowledge and the development of approximate number word knowledge. Previously, it was suggested that cardinal principle knowledge may be a prerequisite for approximate number word knowledge (Le Corre & Carey, 2007). However, our finding that some children have approximate number word knowledge *before* learning the cardinal principle contradicts this theory. Another theory is that approximate number word knowledge actually helps children to learn the cardinal principle (e.g., Wagner & Johnson, 2011). Although it is possible that this is the case for some children, it cannot be the only route to cardinal principle knowledge given that some children learn the cardinal principle *before* developing approximate number word knowledge (Le Corre, 2014; Le Corre and Carey, 2007).

A third possibility, which we believe to be the most promising given the current studies, is that the developmental trajectories for exact number word knowledge and approximate number word knowledge are *not* causally related. The fact that older children—whether subset-knowers or CP-knowers—were better than younger children at saying larger number words when shown larger set sizes suggests that this ability proceeds from cognitive capacities that increase with age such as executive function, count list fluency, associative mapping ability, spatial skills, and non-verbal ANS acuity (e.g., Davidson et al., 2012; Halberda and Feigenson, 2008; Sullivan and Barner, 2014; Verdine et al., 2014). The lack of relation between approximate number word knowledge and children's knower-levels also suggests that the experiences, skills, and cognitive systems underlying these two aspects of numerical development may be quite distinct. This is consistent with previous work suggesting that the parallel-individuation system supports children's learning of exact cardinal number words, whereas the ANS supports verbal numerical approximation (e.g., Carey, 2009; Feigenson et al., 2004).

The current studies have several limitations. One limitation is that our studies were designed to examine whether children responded with higher number words to larger set sizes (or vice versa, referred to as "approximate number word knowledge" or "mapping") but were not designed to examine scalar variability in responses. Because we did not assess scalar variability, it remains possible that children's successful performance (responding with higher number words to larger set sizes) may reflect knowledge that later numbers in the count list denote larger cardinalities (i.e., a "later is greater" rule) rather than a mapping of number words to the ANS. Although the current studies cannot prove that the results reflect a mapping of number words to the ANS, we believe that this is the most plausible explanation given that previous studies using the same tasks have shown that children who respond with higher number words to larger set sizes (and vice versa) also show signatures of the ANS (Le Corre and Carey, 2007; Wagner and Johnson, 2011).

Another limitation is that our verbal comprehension task (Give-*N*) in both Studies 1 and 2 included very few trials for the large set sizes (typically 1 trial for each set size 6 and 9). This likely led to higher variability in our estimates of children's slopes, which may have reduced the chance of finding evidence for approximate number word knowledge in the high-number range (above 4). This may also help to explain why children in Study 2 failed to show approximate number word knowledge on the Give-*N* task but did show such knowledge on the Fast Dots task. Further research is needed to better understand older subset-knowers' approximation ability on verbal comprehension tasks like Give-*N*.

A final, and important, limitation is that Studies 1 and 2 were not designed to be directly compared and featured several procedural changes that, although intended as improvements, limit our ability to compare across studies. Most notably, the Fast Cards task (Study 1) and Fast Dots task (Study 2) differed in multiple respects. For example, in the Fast Dots task, children were shown a larger overall range of set sizes (1–51) than in the Fast Cards task (1–9), potentially helping children to understand that they should think approximately or making children more comfortable saying large number words. In addition, on the Fast Dots task (but not the Fast Cards task), the experimenter modeled the words "eight," "fifteen," "thirty," and "fifty," which may have primed children to use these larger number words. However, in post hoc analyses where we pooled the data across studies, the study children were in (Study 1 vs. Study 2) did not predict their performance, suggesting that these task differences were not critical, whereas child age was strongly related to children's verbal estimation skills. Thus, we believe that differences in child age (older children in Study 2 than in Study 1) provide the most plausible explanation for children's better verbal estimation in Study 2 than in Study 1. Nevertheless, the procedural differences between Study 1 and Study 2, as well as the background of conflicting results in prior studies (Le Corre and Carey, 2007; Wagner and Johnson, 2011), highlight the need to replicate these findings, especially the novel result that age, but not knower-level, predicts mapping ability among subset-knowers. A careful replication involving subset-knowers from a wide age range, using tasks that are carefully matched on procedural factors (e.g., the set sizes presented, trials per set size, modeling by the experimenter), is needed to fully confirm the results presented here.

In summary, the results of two studies indicate that subset-knowers do have some knowledge of the magnitudes represented by number words above their knower-level. In a sample of young (ages 3.0–4.2 years) subset-knowers in Study 1, this knowledge appears to be limited to the difference between the numbers just above their knower-level (up to 4) and all other higher numbers. However, in an older group of subset-knowers in Study 2 (ages 3.1–5.6 years), the children as a group—and especially the older children—showed approximate number word knowledge on the verbal production task for numbers up to 10. Thus, subset-knowers *can* show approximate number word knowledge for larger set sizes, ruling out the possibility that children must learn the cardinal principle prior to gaining this knowledge. Furthermore, across both studies, subset-knowers' approximate number word knowledge was related to their age but not their knower-level, suggesting that approximate number word knowledge and exact number word knowledge may develop through somewhat independent processes proceeding in parallel.

It will be important for future research to delineate the cognitive skills, such as spatial visualization, count list fluency, executive function, and ANS acuity (e.g., Verdine et al., 2014), and adult inputs, such as number talk (e.g., Gunderson and Levine, 2011), that contribute to the development of approximate number word knowledge and to determine whether these differ from those that contribute to exact number word knowledge. Although it is possible that there is overlap in the skills and inputs that are important for exact number word knowledge and approximate number word knowledge, our finding of a lack of correlation between knower-level and mapping ability suggests that the relevant skills and inputs may differ in each case. Given the importance of numerical approximation for later math achievement (e.g., Booth and Siegler, 2008; Bugden and Ansari, 2011; Sasanguie et al., 2013), understanding how to foster the development of this skill may ultimately lead to interventions that can improve children's early number knowledge as well as their long-term math achievement.

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