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Preschool Children's Mathematical Knowledge: The Effect of Teacher "Math Talk"

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This study examined the relation between the amount of mathematical input in the speech of preschool or day-care teachers and the growth of children's conventional mathematical knowledge over the school year. Three main findings emerged. First, there were marked individual differences in children's conventional mathematical knowledge by 4 years of age that were associated with socioeconomic status. Second, there were dramatic differences in the amount of math-related talk teachers provided. Third, and most important, the amount of teachers' math-related talk was significantly related to the growth of preschoolers' conventional mathematical knowledge over the school year but was unrelated to their math knowledge at the start of the school year.

Keywords: early math development, preschool mathematics, teacher input, number skills

By the start of kindergarten, children demonstrate wide individual differences in their mathematical knowledge. Whereas some children have an impressive array of mathematical skills—including the ability to count the number of elements in small sets, to match sets on the basis of their cardinality, to order sets in terms of numerosity, and to carry out simple calculations—others evince much less skill (e.g., Baroody, 1987; Brannon & Van de Walle, 2001; Bullock & Gelman, 1977; Clements, 2004; Gelman, 1972; Huttenlocher, Jordan, & Levine, 1994; Levine, Jordan, & Huttenlocher, 1992; Mix, Huttenlocher, & Levine, 2002; Wynn, 1990). On average, young children from middle socioeconomic status (SES) families have higher levels of mathematics achievement than their lower SES peers (e.g., Jordan, Huttenlocher, & Levine, 1992; Saxe, Guberman, & Gearhart, 1987). Such early differences have long-lasting implications for later school achievement, becoming more pronounced during elementary school (Case & Griffin, 1990; Denton & West, 2002; Entwisle & Alexander, 1990; Griffin, Case, & Siegler, 1994; Jordan et al., 1992) and continuing on into middle school and high school (Braswell et al., 2001). It is important to note that these early differences in mathematical knowledge are associated with differences in the input children receive (e.g., Blevins-Knabe & Musun-Miller, 1996; Saxe et al., 1987).

The early emergence of individual differences in mathematical knowledge, coupled with the fact that about 70% of children in the United States attend preschool or day care at 4 years of age (U.S. Department of Education, National Center for Education Statistics, 2000), motivated our examination of whether the input variations that occur in preschool and day-care settings contribute to children's early differences in mathematical knowledge. Although we know that early environmental input is important to the development of a wide range of cognitive skills (e.g., Campbell, Pungello, Miller-Johnson, Buchinal, & Ramey, 2001; Huttenlocher, Vasilyeva, Cymerman, & Levine, 2002; Reynolds & Temple, 1998), and that day-care and preschool settings are potentially a significant source of this input, very little is known about the nature and frequency of mathematical input in preschool classrooms or about the effects of such input variations on children's mathematical development.

Existing research on early mathematical input in preschool classroom settings has mostly focused on the effectiveness of enrichment or intervention programs. Not surprisingly, immediately following short-term programs targeting specific mathematics concepts, participating children show greater mastery of these concepts than control children. More impressive is the finding that these gains persist a year or more later (e.g., Arnold, Fisher, Doctoroff, & Dobbs, 2002; Griffin & Case, 1996; Griffin et al., 1994). Other studies have shown that comprehensive early intervention programs have a positive impact on children's math achievement as well as other cognitive and social skills. For example, children enrolled in full-time, high-quality educational child care from infancy through 5 years of age had higher math achievement as late as young adulthood than did control children who did not experience the intervention but who received the same nutritional supplements and social work services provided to the intervention group (Campbell et al., 2001).

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Although these studies demonstrate that young children's mathematical knowledge is influenced by environmental input, they leave several important questions unanswered. First, because most intervention programs aim to increase general intellectual skills, the specific kinds of input that affect young children's mathematical skills remain unclear. Second, the positive effects of intervention programs tell us little about whether naturally occurring variations in the math input provided in preschool classrooms affect children's mathematical development.

The few existing studies investigating the effects of variations in math input on children's mathematical skills have focused on parental input. Parental input has most commonly been assessed by interviewing parents or by having them fill out checklists that include a range of mathematically relevant activities. For example, using a structured interview, Saxe et al. (1987) found that middle-SES mothers engaged their children in more complex number activities than low-SES mothers. Mirroring the differential input provided, the middle-SES 4-year-olds performed better on relatively complex numerical tasks than their low-SES peers. Similarly, Blevins-Knabe and Musun-Miller (1996) asked parents to estimate how often their kindergarten children had engaged in each of more than 30 number-related activities during the previous week. They found that the frequency of children's number-related activities at home was positively correlated with children's numerical knowledge as measured by the Test of Early Mathematics Ability–Second Edition (TEMA-2; Ginsburg & Baroody, 1990). Other studies have examined the interaction of parent–child dyads engaged in prescribed counting and set-matching tasks, which varied in complexity. In general, mothers gave more specific, directed instruction to children at lower levels of competence, and children generally were more successful as a result of this input (e.g., Saxe et al., 1987). Unlike the present study, these studies examined the relation of input to the child's mathematical skill at a particular time point rather than to the growth of children's mathematical skills.

In the present study, we investigated the effects of variations in input from preschool or day-care teachers on the growth of children's mathematical skills. Focusing on teacher input rather than parent input is an important step in helping to separate biological from environmental effects on the growth of mathematical skills because teachers, unlike parents, are not genetically related to children. Nonetheless, it is possible that parents with higher levels of math ability select preschools that provide more and better input in this domain. Further, it is possible that children with higher levels of math knowledge elicit more math input from their teachers. A pattern of results showing that the amount of teacher math input is not related to children's mathematical knowledge at the start of the school year but is related to the growth of their math skills over the school year would challenge those explanations and would lend support to the proposal that teachers' math input propels the growth of children's mathematical knowledge. Such findings would go beyond earlier studies that correlated parental math input and children's mathematical knowledge, studies in which the direction of influence was less clear. Of course, definitive evidence that teacher input is causally related to the growth of children's mathematical knowledge would require an experimental study that randomly assigned children to conditions with different levels of input.

Our primary question was whether the total amount of mathematically relevant input preschool teachers provide in their speech is related to the growth of children's mathematical knowledge over the 4-year-old nursery school year. Although we were interested in the relation of particular types of input to the growth of particular types of mathematical knowledge, our ability to address this question was constrained by the size of the speech samples obtained from teachers and by the type of assessment we gave the children. We do report the relative frequencies of different kinds of input teachers provided, and these data show that certain kinds of math input are much more frequently provided in teacher language than are other kinds.

A number of studies have shown that the overall amount of language input children receive is related to their general vocabulary growth (e.g., Hart & Risley, 1992; Huttenlocher, Haight, Bryk, Seltzer, & Lyons, 1991). Moreover, the specific lexical items acquired appear to be sensitive to variations in amount of input. For example, Hoff and Naigles (1998) reported that the earliest verbs children acquire tend to be those that are most frequent in the input. Such findings led us to hypothesize that the amount of "math talk" children hear will impact their acquisition of mathematically relevant language.

Developmental studies indicate that early quantitative representations are linked to quantitative language, notably, to knowledge of the count words. In particular, children's ability to represent exact number is related to the acquisition of counting skills, especially when sets contain more than a few items or when sets being compared contain dissimilar items (Huttenlocher et al., 1994; Jeong & Levine, 2005; Mix, 1999; Mix, Huttenlocher, & Levine, 1996). Number words might serve to call attention to the fact that sets labeled with the same number word are numerically equivalent (Mix et al., 2002). Wynn (1990) proposed that counting begins as a meaningless game, almost like reciting nursery rhymes, but that children come to abstract important mathematical properties from these experiences with the count words. She reported that children learn the meanings of *one*, *two*, and *three* individually, in ascending sequence, and that only after acquiring these words do they then learn the cardinal word principle and the meanings of all the number words in their counting range. Consistent with these developmental studies are reports from cross-cultural studies that speakers of languages that lack an elaborated counting system have limited ability to represent number exactly (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). These cross-cultural findings, together with the developmental findings, led us to predict an association between the early mathematical development of young children and the amount of their exposure to mathematically relevant language.

Our initial examination of transcripts of teacher speech revealed that quantitative input seemed to occur in several different contexts that had the potential to promote children's mathematical knowledge. These contexts ranged from planned mathematical instruction (e.g., children were asked to count sets of objects, to compare the numerosity of two sets, or to carry out calculations such as "If we have twenty-one children in class and one person is missing, how many people do you think there are at our rug now?") to input that occurred in the context of another activity (e.g., children engaged in an art project that involved constructing a book were asked to put numbered pages in order), to incidental comments about quantity (e.g., "Can you tell me what is different about those two beads?" and "What do we do when we have more than one friend who wants to play with something?"). Although some types of input may be more instructive than others, all of these types of input have the potential to promote the acquisition of mathematical language and concepts. Moreover, it was not always easy to determine whether the quantitative inputs in teacher speech were planned or were more incidental. Thus, we made the decision to include them all in our assessment of the amount of mathematical input a teacher provides through speech.

To our knowledge, the present study is the first to investigate math input by examining transcripts of speech to children, a method that enabled us to capture incidental mathematically relevant input as well as planned instruction and to obtain accurate information about the frequency and nature of such input. Unlike relying on questionnaires or checklists to assess input, this approach allowed us to assess mathematical input as it occurred online rather than relying on adults' memory of what inputs occurred. This may be particularly important for the more incidental instances of mathematical language, both because teachers may not consider these instances as math input and because they may have difficulty remembering such instances. Thus, an added benefit of transcribing and coding teacher speech was that it enabled us to capture all of the teachers' mathematical language. Further, unlike methods in laboratory studies in which child–adult dyads were observed interacting over a prescribed numerical activity, this method allowed us to examine the input that occurred in a more naturalistic manner and thus to gain better information about the amount and nature of mathematically relevant language preschool children hear at school.

We also examined whether any relation between amount of math input and children's math growth over the school year was specifically tied to teachers' "math talk" by including more general aspects of input in our analyses of factors related to children's math growth. These other input measures were general quality of the classroom, as indexed by a measure based on the National Association for the Education of Young Children's (NAEYC) preschool checklist (Hyson, Hirsch-Pasek, & Rescorla, 1990), and syntactic complexity of teachers' speech, a measure that we previously found to be related to children's syntactic development (Huttenlocher et al., 2002). We used hierarchical linear modeling, a form of mixed-model regression analysis, to examine the relation among growth in students' mathematical knowledge and math input, classroom quality, and syntactic complexity of teachers' speech. This analytical technique permitted us to treat classrooms and schools as nested random effects, appropriately recognizing their contributions to the variance of the structure.

Method

Participants

Our sample included children from 26 classrooms drawn from 13 preschools and day-care centers in the Chicago area.[1] As is true nationally, the schedules for preschools and day-care centers are quite variable in the Chicago area; about half of our schools were half-day programs, and the other half were full-day programs. This study was part of a larger project examining the effects of teacher language, and teachers were told that we would audiotape their speech during a typical school day in the middle of the school year and that we would administer a cognitive assessment battery to children in their classes at the beginning and end of the school year. The assessment battery included vocabulary and syntax comprehension tests as well as our math assessment. Teachers did not know that we were particularly interested in the relation of math input to the growth of children's mathematical knowledge.

To recruit classrooms for our study, we sent letters to 20 public and private preschools in Chicago and the surrounding suburbs. School directors were then contacted by phone, and those who expressed interest in participating in the study were included. The participating schools represented a wide range of socioeconomic and ethnic groups. Although we tried to contact a range of schools that were representative of the demographics of the greater Chicago area, because this was not a probability sample, we cannot be certain that our results will generalize.

All classrooms within participating schools that served children in the targeted age range were included provided that at least 3 children in the classroom participated in our study at both assessment time points. All children in these classrooms whose parents returned signed permission forms were included, with one exception: Non-native English speakers were not included in our analyses. Class sizes ranged from 14 to 25 children, but not all children returned permission slips; hence, the number of participating children in each class ranged from 3 to 15, with a median number of 6.

Participating schools were categorized into one of three SES groups: low, middle, or high. The directors of all schools, including the Head Start schools, were called to obtain information about the income and education levels of the families they served, because this procedure was thought to provide more accurate information about SES at the school level than would census tract data. We started by providing directors with median income and education information reported for their areas in the 2000 United States Census tract data. Directors were asked to tell us whether this information was correct for the families their school served, and if it was not, to correct it. The four schools (containing four of our classrooms) in the low-SES groups served families with estimated incomes below $25,000, and less than 25% of the parents had attained a BA or higher degree. The three schools (containing eight of our classrooms) in the middle-SES group served families with estimated incomes of $25,000 to $75,000, and between 25% and 50% of parents had attained a BA or higher degree. Finally, the six schools (containing 14 of our classrooms) in our high-SES group served families with estimated incomes above $75,000, and more than 70% of these parents had attained a BA or higher degree. Reflecting the demographics of the Chicago area, some of the schools served primarily Caucasian children, some served primarily African American children, and some served children from a variety of ethnic/racial groups.

We assessed the math skills of a total of 198 children across the 26 classrooms in the study. Most children were tested twice, once at the beginning and once at the end of the school year (i.e., October and April). However, 52 children were absent when one of the assessments was carried out or had moved or changed programs by the end of the school year. When these children were eliminated, 146 of the 198 children remained in the sample. Of these, 6 scored at the ceiling level at the first testing time point, and because these children could not show growth of mathematical knowledge, they were excluded from our analyses, which left 140 children. On average, the children were 4 years 8 months old (SD = 4.5 months) at the first testing point and 5 years 2 months old (SD = 4.5 months) at the second testing point. Boys and girls were roughly evenly represented in the sample. We also audiotaped the speech of the 26 head teachers for later transcription and coding of math input.

[1] The classrooms in the present study were drawn from the same schools as those included in a study of the relation of complexity of teachers' syntax to children's syntactic comprehension (Huttenlocher et al., 2002). However, the data for this study were collected during the 4-year-old preschool year, whereas the data for Huttenlocher et al.'s study were collected during the 3-year-old preschool year. The same sample of teacher speech was used to code both math input and syntactic complexity of speech.

Procedure

Assessment of children's mathematical knowledge. Children's math knowledge was evaluated at the beginning (October) and end (April) of the school year using an assessment consisting of 15 questions preceded by two sample questions. Children were assessed individually in a quiet place in their preschools during a normal school day. Each testing session lasted approximately 10 min. Alternative forms of the assessment were used in the fall and the spring in order to minimize practice effects, with the order of forms counterbalanced across children. Pilot testing showed that the alternative forms were equivalent in difficulty, and our reassessment of this equivalence after the fall testing concurred with this pilot testing.

Each question on the 15-item assessment was presented in a multiple-choice format. The following kinds of knowledge were assessed: ordinality, cardinality, calculation, shape names, understanding "half," and recognizing conventional number symbols. For ordinality and cardinality, sets of different numerosity were used in order to vary difficulty (see Table 1 for more detail on items). Success on these questions depended on the child's knowledge of mathematical concepts and the corresponding language of mathematics, for example, the concept of ordinality as well as the number words and quantitative terms that express this concept. Our earlier work showed that middle-income children outperformed low-income children on verbally presented arithmetic calculations (both word problems and number fact problems) but that these groups did not differ on parallel problems presented in a nonverbal format (Jordan et al., 1992; Jordan, Levine, & Huttenlocher, 1994). This finding is consistent with Ginsburg and Russell's (1981) claim that certain mathematical skills (such as those tested in the nonverbal calculation task) develop in a "robust fashion" not dependent on SES-related environmental influences. Such findings suggest that conventional mathematical knowledge may be more sensitive to input variations than nonverbal mathematical knowledge.

We used our 15-item math assessment rather than a standardized test such as the TEMA-2 (Ginsburg & Baroody, 1990) for two reasons. First, our time with each child was limited by the fact that children were also given vocabulary and syntax comprehension tests. Second, the questions on our math assessment were all multiple choice in format, as were the items on the syntax task. We expected that this consistency in item format would make the tasks easier and quicker to administer and that they would yield more reliable data. Split-half reliability of our math assessment (based on a comparison of scores on odd and even items) was significant at both time points (Time Point 1: $r = .493$, $p < .01$; Time Point 2: $r = .439$, $p < .01$). It should be noted that the items on our brief math assessment had a high degree of overlap with those on the TEMA-2. In particular, 13 of 15 items on our assessment tap knowledge of math concepts that are also assessed on the TEMA-2. Thus, we have reason to believe that our short math assessment is both reliable and valid.

Assessment of teacher input. To obtain a measure of teachers' input to children, an observer visited each participating classroom for 2.5 to 3 hours, in January or February of the school year. The head teacher wore a lapel microphone so that his or her speech could be audiotaped. One hour of the tape was later transcribed and coded for mathematically relevant input. In an attempt to gather input from comparable situations across the different classrooms, the hour selected for transcription included "circle time" and the time immediately following circle time. *Circle time* is a time in which the entire class gathers to participate in discussions, to receive instruction, to sing, and so forth; most preschool and day-care programs include some version of circle time in their daily schedule. By using circle time to assess math input in all classrooms, we eliminated random variation in input that occurs around different contexts. We also succeeded in using a time during which teachers have the potential to engage in math talk and during which all children in the class have access to this talk. Although observing teacher talk on only one occasion leads to measurement error, our observations suggested that activities during this time were routinized and familiar to children. Transcriptions included only the teachers' speech and occasional notes explaining the context. Our method of coding teachers' speech for mathematically relevant input is described in detail below.

In addition to audiotaping teacher speech, the observer filled out a questionnaire based on the NAEYC checklist for preschools (Hyson et al., 1990). We included 10 questions that assess the general quality of teaching, for example, the extent to which teachers attempt to involve children in activities by stimulating their curiosity and interest, the extent to which teachers use redirection or positive reinforcement as discipline techniques, and so forth. For each question, the observer assigned a score that varied from 1 (*not at all like this classroom*) to 5 (*very much like this classroom*). The average score was calculated over the 10 questions and used as our measure of general classroom quality.

Analysis of transcripts. Within 24 hours of a classroom observation, the observer who visited the classroom transcribed the audiotape and provided context notes to assist coders in interpreting the input. Coders identified instances of math input from transcriptions of the audiotapes. The following nine types of input were coded as instances of mathematically relevant input (see Table 2 for examples of each input type):

1. *Counting* encompassed both reciting counting words and counting objects in sets because it was not possible to differentiate these input types on the basis of audiotapes.

2. *Cardinality* involved stating (or asking for) the number of things in a set without counting them. If cardinality was used to reinforce counting, it was coded as a separate instance, for example,

Table 1
Assessment Items

Item type	Instruction	Choices
Ordinality	"Point to the one that has more."	7 dots, 5 dots
Cardinality	Written symbol: Child was shown card with the number "2" on it and was asked, "Which one of these goes with this one?"	1, 2, 3, 4 dots
	Oral number: "Point to four."	2, 3, 4, 5 objects
Calculation	"Johnny has one apple and his mommy gives him one more. Point to how many apples Johnny has now."	1, 2, 3, 4 apples
Shape names	"Point to the triangle."	Triangle, square, circle, rectangle
Understanding "half"	"Point to the one that shows half."	1/2, 1/4, 1/3, 2/3 of shaded circle
Recognizing conventional number symbols	"Point to the number."	B, e, 2, *

Table 2
Examples of Types of Math Input Provided by Teachers

Type of input	Examples
Counting	"I shouldn't hear any tap 'til he says one, two, three."
	"Now we counted out 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 teeth to go in the top of your mouth."
	"Alright Dana, help us count: 1, 2, 3, 4, 5, 6, . . . 21, 22 Tommy 23." [counting days on the calendar]
Cardinality	"Sure, all three of you can help me."
	"But how many crocodiles did he have?"
	"Can you tell me what's different about those two beads?"
Equivalence	"We share by dividing equally."
	"Let's see Chris, we had two groups of ten here but it looks like. . .."
	"Okay this and this is the same amount of money. They are the same."
Nonequivalence	"Oh no, you have more than twelve teeth."
	"What do we do when we have more than one friend that wants to play with something?"
	"Seven people said yes, ten people said no. Which one was more people?"
Number symbols	"OK, right up here at the top, what number do I have?"
	"OK Jacqueline, can you get the blue nine?"
	"Good, OK, if you have a seven put a dinosaur on it." [*Dinosaur Bingo* game]
Conventional nominative	"We'll read part two about Israel about what the children are wearing."
	"Not February the sixth, today is not February sixth."
	"Remember a little while ago when we did the play *The Three Little Pigs?*"
Ordering	"Very good, yesterday was seventy-four and today is number seventy-five."
	"Nine, what comes after nine?"
	"Tuesday was twenty-two. Alex what do you think Wednesday is?"
Calculation	"We're going to count out ten beans for your top teeth and ten beans for your bottom teeth, which makes twenty."
	"And if you take three away from six how many will you have?"
	"If we have twenty-one children in our class and one person is missing, how many people do you think there are at our rug now?"
Placeholding	"How are we going to count to seventy-five, John? . . . Are we going to use ones or tens and ones?"
	"How many times are we going to count by ten? Seven times, how many ones? Five."

"One, two, three. There are three books." would be coded as two instances, one of counting and one of cardinality.

3. *Equivalence* encompassed statements describing a quantitative match, either of number or of amount, between two or more entities. These included (a) one-to-one mapping (e.g., each child gets one cracker), (b) one-to-many mapping (e.g., each group has four children), and (c) stating that two amounts or sets are the same.

4. *Nonequivalence* encompassed statements of two or more entities being unequal, whether referring to (a) unspecified amounts (e.g., "Who has the most?"), (b) one amount specified and the other(s) unspecified (e.g., "Oh no, you have more than 12 teeth"), or (c) all relevant amounts specified (e.g., "Seven people said yes, 10 people said no. Which one was more people?").

5. *Number symbols* were coded if utterances included instances in which a teacher labeled a written number symbol or asked a child to identify, write, or find a number symbol (e.g., "3" in a stack of cards with printed numbers).

6. *Conventional nominatives* used numbers as labels for things or dates.

7. *Ordering* instances referred to a sequence and made explicit reference to more than one entity or set. Note that reciting a list of number words in order would not be coded as ordering but rather as counting.

8. *Calculation* instances included cases in which a teacher performed a calculation or asked a child to solve a calculation problem.

9. *Placeholding* encompassed any input that referred to place value: ones, tens, hundreds, and so forth and included, but was not limited to, the decomposition of (at least) two-digit numbers.

Reliability was established on 35% (9/26) of the transcripts. The reliability for the total amount of teacher input and for each of the categories of input was high (total amount of input: $r = .99$; range for each input category: $r = .81$–.99). It was not possible to establish reliability for calculation or placeholding, as these input types were rare and did not occur in the nine transcripts used for reliability coding.

In order to address our main question of whether the overall amount of math input provided by teachers is related to the growth of children's math skills, we computed the total number of instances that occurred for each teacher across the nine types of input coded. As described above, a single utterance often contained more than one instance of math input. For example, a teacher might say, "I see three trucks in the block area and four trucks in the kitchen area, so how many trucks are there altogether?" According to our coding system, this statement would contain three instances of math input: two instances of cardinality ("three trucks" and "four trucks") and one instance of calculation ("how many trucks are there altogether?"). Most utterances that contained instances of equivalence, nonequivalence, ordering, calculation, or placeholding also contained instances of cardinality.

Analysis. Our design involved a sample of students nested within classrooms which, in turn, were nested within schools. Both classrooms and schools were taken to be random effects because the specific classrooms and schools in our sample were not of particular interest in and of themselves but were of interest only because they represented samples from populations of classrooms and schools. In our examination of SES effects on math input, our only independent variable was categorical, so we used a nested analysis of variance (ANOVA) model to analyze these data. However, we were also interested in the relation of quantitative independent variables (math input, syntax input, and classroom quality) and math knowledge gains in the context of this nested design. Given the structure of the design, ANOVA was not appropriate because the independent variables were not categorical. Conventional multiple regression analysis was likewise not appropriate because it assumes that the design is a simple random sample (without nesting). Application of multiple regression methods in cases of nested designs violates assumptions of the statistical methods and leads to significance tests that are inaccurate (see Raudenbush & Bryk, 2002). An alternative analytic strategy that is appropriate for nested designs is based on hierarchical linear models (HLM; see Raudenbush & Bryk, 2002). Analyses using HLM provide accurate significance tests for association as well as estimates of the variance components for the random effects associated with the design.

In the design of this study, students were nested within classrooms, which were nested within schools, where both classrooms and schools were random effects. The effects of math input, teacher syntax, and classroom quality occurred at the level of the classroom, but the effect of school SES occurred at the school level. To analyze data with two nested random effects, we used a three-level hierarchical linear model (Raudenbush & Bryk, 2002). The analytic model can most easily be described in terms of the statistical model at each of the three levels of the analysis. At the first level (students within classrooms), the model for the gain in mathematics knowledge for the kth student in the jth classroom in the ith school (denoted Y_{ijk}) is

$$Y_{ijk} = \pi_{0ij} + \varepsilon_{ijk},$$

where π_{0ij} is the average gain in mathematics knowledge in the jth classroom in the ith school and ε_{ijk} is a student-specific residual.

We were interested in understanding the relation among characteristics of math input, teacher syntax, and classroom quality, all of which occur at the level of the classroom. Thus we used Level 2 (classroom-level) models that included these input characteristics. We actually fit two slightly different kinds of Level 2 models. One of them included the three characteristics one at a time. The other included these characteristics simultaneously. Specifically, the Level 2 model including all three characteristics simultaneously is

$$\pi_{0ij} = \beta_{00i} + \beta_{01i}\text{MATHINPUT}_{ij} + \beta_{02i}\text{SYNTAX}_{ij} + \beta_{03i}\text{CLASSQUALITY}_{ij} + \xi_{0ij},$$

where β_{00i} is a school-specific intercept, β_{01i} is the relation between math input and class mean mathematics knowledge gain in school i, β_{02i} is the relation between syntax input and class mean mathematics knowledge gain in school i, β_{03i} is the relation between classroom quality and class mean mathematics knowledge gain in school i, MATHINPUT_{ij} is the math input score of classroom j in school i, SYNTAX_{ij} is the syntax input score of classroom j in school i, CLASSQUALITY_{ij} is the classroom quality score of classroom j in school i, and ξ_{0ij} is a classroom-specific residual (random effect). The models that included the characteristics individually were similar except that they each had only one of the three predictors at Level 2.

We also expected that achievement gains would vary across schools (which are random effects). The sample size was not large enough to simultaneously estimate the variance of all of the possible input effects across schools (although exploratory analyses suggested that these variations were not large). Consequently, the Level 3 (school-level) model is

$$\beta_{00i} = \gamma_{000} + \gamma_{001}\text{SES}_i + \eta_{00i}$$

$$\beta_{01i} = \gamma_{010}$$

$$\beta_{02i} = \gamma_{020}$$

$$\beta_{03i} = \gamma_{030},$$

where γ_{000} is the average intercept across schools, γ_{001} is the association of average school SES on school average gains in math knowledge, γ_{010} is the average relation between math input and class mean mathematics knowledge gain across schools, γ_{020} is the average relation between syntax input and class mean mathematics knowledge gain across schools, γ_{030} is the average relation between classroom quality and class mean mathematics knowledge gain across schools, and η_{00i} is a school-specific random effect.

Thus the object of the statistical analysis was to estimate the five fixed effects ($\gamma_{000,}$ γ_{001}, γ_{010}, γ_{020}, and γ_{030}) and the three variance components—one at each level of the design (the person-specific variance of ε_{ijk}, the classroom-specific variance of ξ_{0ij}, and the school-specific variance of η_{00i}).

We also examined the three input characteristics (math input, syntax input, and classroom quality) separately. In these analyses, the Level 2 model differed in that it included only one of the inputs, but the Level 1 and Level 3 models were identical to those given above.

Results

Children's Math Scores

We first examined individual children's math scores at the beginning and end of the school year. Proportion correct responses on the math assessment were calculated at each time point for those children present at both the fall and spring assessments who were not at ceiling at the fall test point (140 of the total 198). These scores were then arcsine transformed for statistical analyses, as is standard when data are reported as percentages.

An analysis of variance with SES of the school's population (low, middle, high) as a between-subjects variable and assessment time (fall, spring) as a within-subject variable revealed main effects of assessment time, $F(1, 137) = 33.255$, $p < .01$, and SES, $F(2, 137) = 28.906$, $p < .01$, and no interaction, $F(2, 137) = 1.139$, $p = .323$ (see Figure 1). The main effect of assessment time reflected better performance in the spring ($M = .82$, $SD = .14$) than in the fall ($M = .71$, $SD = .17$), Cohen's $d = .88$. Tukey's honestly significant difference (HSD) tests revealed that the average score of children at schools serving low-SES families ($M = .55$, $SD = .15$) significantly differed from the average scores of both those at schools serving high-SES families ($M = .80$, $SD = .15$), $p < .01$, Cohen's $d = 1.41$, and those at schools serving middle-SES families ($M = .79$, $SD = .13$), $p < .01$, Cohen's $d = 1.27$. Children at high- and middle-SES schools did not differ significantly from each other ($p > .60$). Although the absence of a significant School SES × Assessment Time interaction might be interpreted as showing comparable growth of math knowledge in the three SES groups, the amount of growth shown by children in the higher SES groups might have been constrained by ceiling effects at the second assessment time point; of the 16 children who were at ceiling at the posttest, 14 attended schools serving high-SES families and 2 attended schools serving middle-SES families.

Teacher Input Measures

Teacher math input in the 26 classrooms varied widely, both in terms of the amount of input and the diversity of input types provided. The amount of math input teachers provided ranged from 1 to 104 instances during the hour of input we coded; the

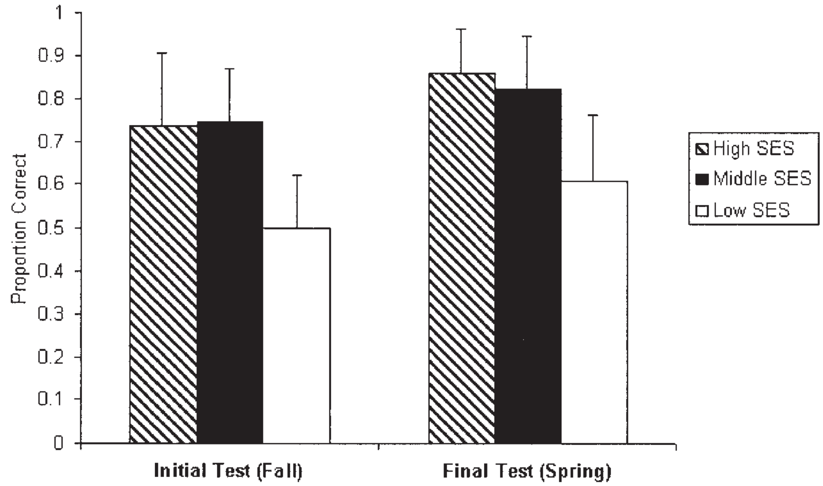

Figure 1. Proportion correct on math assessment for the three socioeconomic status (SES) groups (high, middle, and low) in the fall and the spring of the 4-year-old preschool year.

average number of instances was 28.3 (SD = 24.2). The diversity of types of math input also varied widely, with teachers providing between 1 and 9 different types of input (out of a possible 9 coded types). The average number of math input types provided was 3.9 (SD = 1.8). Not surprisingly, amount and diversity of math input were significantly correlated ($r = .70$, $p < .01$), indicating that teachers who provided a larger amount of math input also tended to provide a greater diversity of types of math input.

Table 3 summarizes the relative frequencies of the different kinds of math inputs in teacher speech. By far the most common type was cardinality—labeling the numerosity of a set (48%). This was followed by labeling written number symbols (17%), counting (13%), and conventional nominatives (9%). The four most frequent categories combined accounted for 87% of all inputs. The other five input types (calculation, ordering, nonequivalence, equivalence, and placeholding) each accounted for 5% or less of the total inputs and occurred in less than half of all the classrooms in our sample.

An analysis of variance examined whether the amount of math input provided in teacher speech differed significantly across classrooms serving different SES groups. The effect of SES was not significant ($F = 1.23$, $p = .312$, *ns*). Of note, amount of teacher math input was not correlated with mean classroom math scores at the fall testing time point ($r = .001$, $p = .996$, *ns*). Thus, it does not appear to be the case that teachers provided different amounts of math input in response to children's initial level of mathematical knowledge or as a function of their SES. The same pattern of results was found for diversity of kinds of input. Because parallel results were obtained on amount of input and diversity of input in all our analyses, in subsequent analyses we report only results for amount of input.

General classroom quality was assessed using a measure based on the NAEYC checklist (maximum average score across questions was 5). An analysis of variance revealed a main effect of SES, $F(2, 23) = 7.84$, $p < .01$. Post hoc Tukey's HSD tests showed that classrooms serving children from high-SES families had higher scores (M = 4.6, SD = 0.46) than both classrooms serving children from middle-SES families (M = 3.8, SD = 0.83), $p < .05$, Cohen's d = 1.97, and classrooms serving children from low-SES families (M = 3.4, SD = 0.29), $p < .01$, Cohen's d = 1.24. The average scores of the classrooms serving middle- and low-income families did not differ significantly from each other ($p = .47$). General classroom atmosphere scores were positively, but not significantly, correlated with math input scores ($r = .25$, $p > .20$). Similarly, the amount of math input teachers provided was positively, but not significantly, correlated with the syntactic complexity of teachers' speech ($r = .18$, $p > .30$; see Huttenlocher et al., 2002, for a description of how syntactic complexity was computed). These findings suggest that the amount of math input teachers provide is somewhat independent of other positive aspects of teacher input.

Table 3
Frequency of Math Input Types: Total Across Classrooms

Input type	Raw number across classrooms	Proportion of total inputs	Number of classrooms using this input type
Cardinality	356	.48	26
Number symbols	123	.17	13
Counting	95	.13	18
Conventional nominatives	67	.09	17
Calculation	36	.05	4
Ordering	29	.04	8
Nonequivalence	15	.02	9
Equivalence	12	.02	7
Placeholding	4	.01	1

Relation of Teachers' Input and Children's Math Growth

The results of the HLM analyses examining the individual effects of each of the three input variables on math knowledge gains are reported in Table 4. In all of our HLM analyses, we multiplied the number of math inputs teachers provided by 20, which corresponds to an estimate of the number of mentions that

Table 4
Results of the Hierarchical Linear Model Analysis Using Predictors Individually

Final estimation of fixed effects	Coefficient	*SE*	*t*	*df*	*p*	95% confidence interval	
						Lower	Upper
Intercept, γ_{000}	.097	.018	5.437	12.000	.000	.058	.136
Math input, γ_{010}	.031	.010	3.299	11.000	.008	.008	.054
Syntax input, γ_{020}	−.084	.231	−0.362	11.000	.724	−.592	.425
Classroom quality, γ_{030}	.046	.023	2.010	11.000	.069	−.004	.096
Socioeconomic status, γ_{040}	.012	.022	0.531	11.000	.606	−.036	.059

would occur in a month if the rate of input observed represents a consistent estimate of input over time. It is important to know that this scale factor has no effect on the significance of our results but makes the numerical results easier to interpret. This table shows that there is a statistically significant association of math input with gains in math knowledge but no statistically significant effect of SES when math input is controlled. The table also shows that there is no statistically significant association between syntax input, classroom quality, or school SES on math knowledge gains. In these analyses, neither the Level 2 (classroom-level) nor Level 3 (school-level) variance components were statistically significant ($p > .50$). We report effect sizes in the form of coefficients of the HLM analysis and their 95% confidence intervals.

The results of the HLM analyses examining all three input variables together are given in Table 5, which shows that there is a statistically significant association of math input with gains in mathematics knowledge but no association of syntax input, classroom quality, or SES with gains in mathematics knowledge when math input is controlled. In this analysis, as in the previous analyses, neither the Level 2 (classroom-level) nor Level 3 (school-level) variance components were statistically significant ($p > .50$). Again, we report effect sizes in the form of coefficients of the HLM analysis and their 95% confidence intervals. The effect size of teacher math input can be evaluated as follows: As previously mentioned, the range of teacher input at a single observation was about 100 (1–104). An increase of 25 mentions per observation period (about a quarter of the range) would be associated with a change in achievement gain of $25 \times 0.00844 = 0.21$ standard deviations of achievement gain, which is in the range of what Cohen (1977) called a small effect; an increase of 50 mentions per observation period (about half the range) would be associated with a change in achievement gain of $50 \times 0.00844 = 0.42$ standard deviations of achievement gain, close to what Cohen (1977) called a medium effect.[2]

Several of the children in our study obtained perfect scores (scored at ceiling) on the measure of math knowledge either at the pretest, the posttest, or both. The scores of children who scored at the ceiling at the pretest could logically only stay the same or decrease on the posttest, and therefore no meaningful measure of growth is possible for them. Consequently, we excluded the 6 children who scored at the ceiling at the pretest from all of our analyses. In contrast, children who were not at the ceiling at the pretest could logically exhibit either growth or decline. Children who were not at the ceiling at pretest but reached the ceiling at the posttest exhibited growth, but their growth was probably underestimated. Because some measure (albeit a probable underestimate) of growth was possible for them, we chose to include in our analyses the 16 children who were not at the ceiling at pretest but reached the ceiling at the posttest. In order to determine whether the choice to include children who scored at the ceiling at posttest might have influenced our results, we repeated all of our analyses with these children excluded (e.g., including only children who did not score at the ceiling on either the pretest or the posttest). The results of those analyses were qualitatively equivalent to those presented here. That is, the results of all of the significance tests were the same with and without children who scored at the ceiling on the posttest included in the sample.

Discussion

Three main findings emerged from the present study. First, consistent with previous studies, there were marked individual differences in children's conventional mathematical knowledge by 4 years of age. On average, according to our assessment, level of mathematical knowledge was higher for children from high- and middle-SES backgrounds than for children from low-SES backgrounds. Second, preschool teachers varied dramatically in the amount of math talk they provided. Third, and most important, our results indicate that the amount of preschool teachers' math talk was significantly related to the growth of young children's conventional math knowledge over the course of the school year. Below we discuss each of these findings in turn.

Consistent with previous findings (e.g., Jordan et al., 1994; Saxe et al., 1987), we found differences in math scores among children from different SES groups. These differences remained constant across the school year, as the growth of children's scores did not differ by SES group. This may have been the case because in our sample of schools, there was not a significant difference in the amount of mathematically relevant input provided in classrooms serving children from varying SES backgrounds. The equivalence in math growth in the three SES groups in the face of comparable amounts of mathematically relevant input is consistent with the finding that amount of teacher input was significantly correlated with the average level of math growth in classrooms. However, as mentioned previously, it is possible that ceiling effects on the math assessment used masked an SES effect, which would reflect greater growth in the higher SES groups.

[2] The effect sizes were calculated by dividing the math input coefficient from Table 5 by 20 to obtain the coefficient for a single day ($.026/20 = .0013$). This was then divided by the residual *SD* of the math gain, $s = 0.154$, to get $\beta/s = .0084416$.

Table 5
Results of the Hierarchical Linear Model Analysis Using All Predictors Simultaneously

Final estimation of fixed effects	Coefficient	*SE*	*t*	*df*	*p*	95% confidence interval	
						Lower	Upper
Intercept, γ_{000}	.094	.016	5.923	11.000	.000	.059	.128
Math input, γ_{010}	.026	.010	2.508	9.000	.033	.004	.049
Syntax input, γ_{020}	−.040	.212	−0.188	9.000	.855	−.519	.439
Classroom quality, γ_{030}	.059	.034	1.714	9.000	.120	−.019	.137
Socioeconomic status, γ_{040}	−.040	.033	−1.238	11.000	.242	−.112	.031

Our second main result was the finding of marked variation in the amount of mathematically relevant input provided by different preschool teachers. During the 1 hour of teacher speech that was coded for each classroom, the number of mathematically relevant instances ranged from 1 to 104. It is interesting that the amount of input provided did not significantly differ across classrooms serving children from different SES groups. If we are correct in thinking that these math input differences are consistent over time, the variations found among the teachers in our sample would be likely to result in large differences in amount of input over the course of the school year. Such consistent differences would be likely to result in differences in the growth of math knowledge among children in different classrooms. Alternatively, if the math input differences found reflect random fluctuations among the teachers sampled, no such relationship between input and classroom math growth would be expected.

This brings us to our third and most important finding, that of a significant relation between the amount of math input in teacher speech and the growth of children's math skills over the school year. The finding of an association between amount of talk about math and the growth of children's conventional math knowledge is likely to be a part of the more general relationship between vocabulary growth and amount of language input (Hart & Risley, 1992; Huttenlocher et al., 1991; Weizman & Snow, 2001). It is reasonable that acquiring the vocabulary of mathematics (e.g., mapping number words to set sizes), like vocabulary acquisition more broadly, is related to amount of input. Although acquiring the language of conventional mathematics is only a part of developing math skills, it is an important tool for fostering mathematical thinking. For example, in tasks such as counting sets of objects accurately, recognizing which of two spoken numbers is greater, or calculating the answer to an addition or subtraction problem, knowledge of the conventional order of a string of number words is necessary, although obviously not sufficient, to succeed. In addition, knowledge of the cardinal meaning of the number words is associated with the ability to represent the exact numerosity of sets and to calculate exact answers to calculation problems, particularly when set sizes exceed a few items. Thus, input that helps children learn the language of mathematics also affects their mathematical skills.

Our HLM analyses suggest that the relation between teacher input and children's math growth is specifically related to teachers' *math* input. Although we found a significant relation between growth of children's conventional mathematical knowledge and the amount of math talk provided by the teacher, we did not find a relation between math growth and our other teacher and classroom measures. In particular, math growth was not related to general classroom atmosphere or to syntactically complex utterances in teachers' speech, a nonmath aspect of teacher input. Although measurement error may have masked the relation of these other variables and math growth, we did find a relation between teachers' syntactic complexity and the growth of children's syntax skills in a related study (Huttenlocher et al., 2002). Thus, it is not the case that measurement error precludes the finding of any relationship between these variables and the growth of children's skills. Further, teachers' math input, teachers' syntactic complexity, and general classroom atmosphere were not significantly correlated, suggesting that the amount of math input teachers provide is somewhat independent of other positive aspects of teacher input. Thus, it appears that the relation between teachers' math talk and the growth of children's math skills does not simply reflect a general input effect.

Finally, we were able to rule out a number of alternative explanations for the relation between math input and math growth. Notably, it did not appear to be the case that higher SES families chose preschools on the basis of the math input provided, because amount of math input did not significantly differ for schools serving children from different SES backgrounds. Further, although children who had more math knowledge at the start of the school year might have elicited more math input from their teachers, this did not appear to be the case because the amount of math input provided by teachers was not correlated with children's math scores at the start of the school year.

This study is a first indication that coding of teacher language provides a potentially useful way of measuring mathematical input in preschool classrooms. In future work, we plan to gather larger samples of speech from each teacher participating and to administer children a standardized math assessment that would provide a more comprehensive view of their math knowledge. This would allow us to take the important next step of examining whether particular types of math input more strongly predict the growth of specific types of math skills or overall growth of children's math knowledge. This work would provide more specific information to preschool teachers about how to foster the mathematical knowledge of young children. An additional important step will be to carry out experimental studies in which preschool children are randomly assigned to treatment groups to determine whether there is a causal relation between amount of teacher math talk and the growth of children's math knowledge.

In the meantime, the results of the current study have important theoretical and practical implications. Theoretically, they indicate that amount of teacher math talk is related to children's mathematical skills. We argue that acquiring the language of mathematics is important to the acquisition of mathematical concepts and to the application of these concepts in problem solving. Practically, our findings suggest that preschool teachers may be able to foster the mathematical knowledge of young children by increasing their "math talk." Although this idea is conveyed in the Standards for Grades Pre-K–2 of the National Council of Teachers of Mathematics (2000), to our knowledge, this is the first study to demonstrate an association between amount of math talk and children's math growth. There are many opportunities in preschool classrooms to engage children in conversations that include rich quantitative information. The examples in our transcripts indicate that many preschool teachers naturally incorporate math talk into their daily routines. The message that more talk about math may have the potential to increase children's math skills is a simple one but one that holds promise for increasing the preparedness of large numbers of young children for the challenges they will face in elementary school and beyond.

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